First Name-- December 22, 2014
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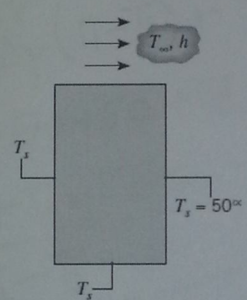
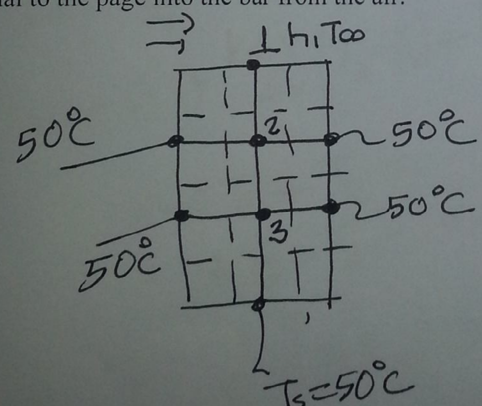
Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer
Midterm Exam III
Closed Notes Open Book
Fall 2014

1) A long bar of rectangular cross section is 60 mm $\times$ 90 mm on a side and has a thermal conductivity of K=1W/ m.K. One surface is exposed to a convection process with air at 100° C and a convection coefficient of H=100W/ m^2 K, while the remaining surfaces are maintained at 50 °C.



Using a grid spacing of 30 mm , determine the nodal temperatures and the heat rate per unit length normal to the page into the bar from the air.



$\Delta X = \Delta Y = 30$ mm

1) $$\frac{K\Delta y/2}{\Delta x}(50-T_1)+\bar{h}\Delta x(T_\infty-T_1)+\frac{\Delta y K}{2\Delta x}(50-T_1)+\frac{K\Delta y}{\Delta x}(T_2-T_1)=0$$

$$\frac{k}{2}(50-T_1)+\bar{h}\Delta x(T_\infty-T_1)+\frac{k}{2}(50-T_1)+k(T_2-T_1)=0$$

$$50-T_1+\underbrace{\frac{\bar{h}\Delta x}{k}}_{Bi}(2)(T_\infty-T_1)+50-T_1+2(T_2-T_1)=0$$

$$50-T_1+2BiT_\infty-2BiT_1+50-T_1+2T_2-2T_1=0$$

$$-4T_1-2BiT_1+2T_2=-100-2BiT_\infty$$

$$-2T_1-BiT_1+T_2=-50-BiT_\infty$$

$$T_1(2+Bi)-T_2=(50+BiT_\infty)$$

$$\boxed{5T_1-T_2=350} \qquad Bi=\frac{h\Delta x}{k}=3$$

node-2

$$\frac{k\Delta y}{\Delta x}(50-T_2)+\frac{k\Delta x}{\Delta y}(T_1-T_2)+\frac{k\Delta y}{\Delta x}(50-T_2)+\frac{k\Delta y}{\Delta x}(T_3-T_2)=0$$

$$-4T_2+50+50+T_3+T_1=0$$

$$-4T_2+T_3+T_1=-100$$

$$\boxed{-T_1+4T_2-T_3=100}$$

Node 3 4

$$\frac{k\Delta y}{\Delta x}(50-T_3)+\frac{k\Delta x}{\Delta y}(T_2-T_3)+\frac{k\Delta y}{\Delta x}(50-T_3)+\frac{k\Delta x}{\Delta y}(50-T_3)=0$$

$$-4T_3+T_2=-150$$

$$4T_3-T_2=150$$

$$\begin{bmatrix}5 & -1 & 0\\ -1 & 4 & -1\\ 0 & -1 & 4\end{bmatrix}\begin{bmatrix}T_1\\T_2\\T_3\end{bmatrix}=\begin{bmatrix}350\\100\\150\end{bmatrix}$$

$T_1=81.6$

$T_2=58.4$

$T_3=52.1$

b)

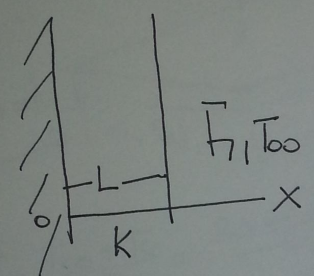
b) $\frac{q}{L} = \bar{h}(\Delta x/2)(T_\infty - 50) + h\Delta x(T_\infty - T_1) + \frac{h\Delta x}{2}(T_\infty - 50)$

$= h\Delta x[(T_\infty - T_{50}) + (T_\infty - T_1)]$

$= 100(0.03)[(100 - 50) + (100 - 87.69)]$

$= 205 \text{ W/m}$

2) Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 125- mm- thick steel plate ($\rho = 7830\,kg/m^3$, c= 550 J/ kg K, k= 48 W/m.K) , which is initially at a uniform temperature of $T_i = 150\,^0C$ and is to be heated to a minimum temperature of $500\,^0C$. Heating is effected in a gas- fired furnace, where products of combustion at $T_\infty = 850\,^0C$ maintain a convection coefficient of h =200 W/ m^2 K on both surfaces of the plate. How long should the plate be left in the furnace?



$$Bi = \frac{hL}{K} = \frac{(200)(0.0625)}{48} = 0.26 > 0.1$$

do not use Lumped capacitance analysis

L = 125 mm

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{500-850}{150-850} = 0.5$$

$$\theta_o^* = C_1 e^{-\xi_1^2 F_o} \quad \text{and} \quad \alpha = \frac{k}{\rho c} = 1.115 \times 10^{-5}\, m^2/s$$

$\xi_1 \cong 0.488 \quad C_1 \cong 1.0396$

$$t = ? \quad -\xi_1^2\left(\frac{\alpha t}{L^2}\right) = \ln\left(\frac{\theta_o^*}{C_1}\right)$$

$$= \ln(0.5/1.0396) = -0.732$$

$$t = \frac{0.732\,L^2}{\xi_1^2\,\alpha} = \frac{0.732(0.0625)^2}{(0.488)^2(1.115\times10^{-5})}$$

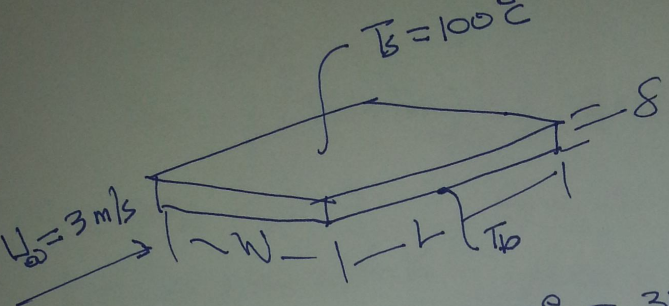
$$= 1080\,s$$

OR Use charts

$$\left.\begin{aligned} \frac{\theta_o}{\theta_i} &= 0.5 \\ 1/Bi &= \frac{1}{0.26} = 3.84 \end{aligned}\right\} F_o \cong 3 \qquad F_o = \frac{\alpha t}{L^2}$$

$$t = \frac{F_o L^2}{\alpha} \approx 1050\,s$$

3) Air at a velocity of 3 m/s and 20 ^{0}C flows over a flat plate along its length .The length, width, and thickness of the plate are 100 cm, 50 cm , 2 cm , respectively. The surface of the plate is maintained at 100 ^{0}C. Calculate the heat lost by the plate and the temperature of the bottom surface of the plate for steady –state conditions. $k = 23\ W/mK$



$\delta = 2\ cm$
$L = 100\ cm$
$W = 50\ cm$

$$T = \frac{T_\infty + T_s}{2} = \frac{20+100}{2} = 60°C = 333K$$ air properties;

$\rho \simeq 1.06\ kg/m^3$ $\quad k = 0.02897\ W/mK$
$\nu \simeq 18.97 \times 10^{-6}\ m^2/s$ $\quad Pr = 0.696 \quad C_p = 1.005\ \frac{kJ}{kg K}$

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(3)(1)}{18.97 \times 10^{-6}} = 1.58 \times 10^5$$ laminar flow

$$\overline{Nu_L} = 0.664 \sqrt{Re_L}\, Pr^{1/3} = (0.664)(\sqrt{1.58 \times 10^5})(0.699)^{1/3}$$
$$= 234.37$$

$$\bar{h} = \frac{k}{L}(\overline{Nu_L}) = (0.02897/1)(234.37)$$
$$= 6.78\ W/m^2 K$$

$$q = \bar{h} A (T_s - T_\infty) = (6.78)(1)(0.5)(100-20)$$
$$= 271.5\ W$$

b) $$q = \frac{KA(T_s - T_b)}{L}$$

$$T_b = T_s + \frac{q\delta}{KA} = 100 + \frac{(271.5)(2/100)}{(23)(0.5)} \simeq 100.5°C$$

4) A rectangular tube, 30 mm by 50 mm, carries water at a rate of 2 kg/s. Determine the length of tube required to heat the water from 30 ^{0}C 50 ^{0}C if the tube wall temperature is maintained at 90 ^{0}C. Hint: Noncircular tubes one can use hydraulic diameter .

$T_{mi} = 30°C$

$T_{mo} = 50°C$

$\overline{T_{mo}} = \frac{1}{2}(T_{mi} + T_{mo}) = \frac{1}{2}(30+50) = 40°C = 313\,K$

$\rho \approx 992\ kg/m3$ $\qquad c_p = 4.174\ J/kg\cdot K$

$k = 0.634\ W/mK$ $\qquad \mu = 6.531\times10^{-4}\ kg/m.s$

$A = (0.03)(0.05) = 0.0015\ m^2$

$P = 2(0.03 + 0.05) = 0.16\ m$

$D_H = \frac{4A}{P} = (4)(0.0015)/0.16 = 0.0375\ m$

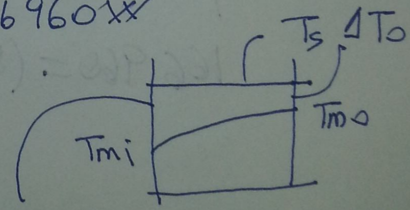
$A_s = PL = (0.16)(L)$

surface area

$q = \dot{m} c_p (T_{mo} - T_{mi})$

$= (2)(4.174)(50-30) = 166960\ W$

$q = \bar{h} A_s \Delta T_{LMTD}$



$\Delta T_{LMTD} = \frac{\Delta T_i - \Delta T_o}{\ln(\Delta T_i / \Delta T_o)}$

$= \frac{60-30}{\ln(60/30)} = 49.32°C$

$\Delta T_i = T_s - T_{mi} = 90 - 30 = 60$

$\Delta T_o = T_s - T_{mo} = 90 - 60 = 30$

$$\dot{m} = \rho V A_c \Rightarrow V = \frac{2}{(992)(0.0015)} = 1.343\ m/s$$

$$Re_{D_H} = \frac{\rho V D_H}{\mu} = \frac{992)(1.343)(0.0375)}{6.531 \times 10^{-4}} = 76557$$

$Re_{D_H} > 2300$ turbulent flow

$$Pr = \frac{\mu c_p}{k} = \frac{6.531 \times 10^{-4} \times 4174}{0.634} = 4.3$$

Use any equation you choose

$$\overline{Nu}_{D_H} = \frac{\bar{h} D_H}{k} = 0.023\ Re_{D_H}^{0.8}\ Pr^{0.4}$$

water is heated

$n = 0.4$

$$= 332.8$$

$$\bar{h} = \frac{k}{D_H}(Nu_{D_H}) = \frac{0.634}{0.0375}(332.8) = 5628\ W/m^2K$$

$$q = \bar{h} A_s \overset{\Delta T}{LMTD}$$

$$166\,960 = (5628)(0.16L)(49.32)$$

$$L = 3.76\ m$$