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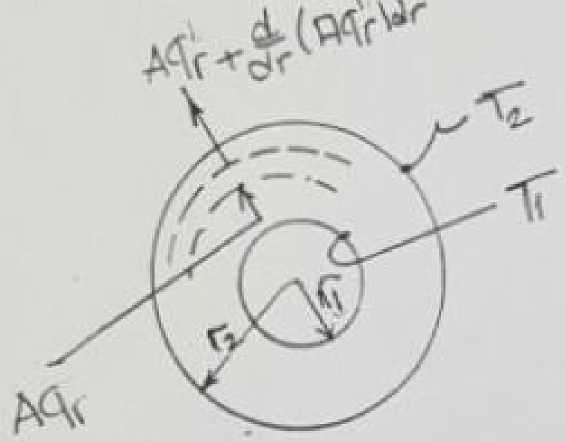
The Hallow Sphere

The hallow sphere with specified Surface temperatures



1- steady state
2- constant properties
3- one-dimensional heat flow
4- no heat generatio

$$\frac{d}{dr}(Aq_r'') = 0$$

$$\frac{dq}{dr} = 0$$

where $q'' = \frac{q}{A}$

$$q = -kA\frac{dT}{dr}$$

$$A = 4\pi r^2$$

$$q = -4\pi k r^2 \frac{dT}{dr}$$

or separating variables

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$$\frac{q}{4\pi k}\int_{r_1}^{r}\frac{dr}{r^2} = -\int_{T_1}^{T} dT$$

$\therefore$

$$\frac{q}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r}\right) = (T_1 - T)$$

and

$$\frac{q}{4\pi k}\int_{r_1}^{r_2}\frac{dr}{r^2} = -\int_{T_1}^{T_2} dT$$

$$\frac{q}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = (T_1 - T_2)$$

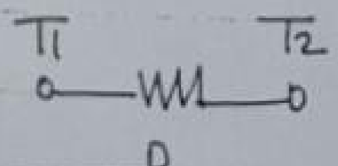
$\therefore$

$$\frac{T_1 - T(r)}{T_1 - T_2} = \frac{\frac{1}{r_1}-\frac{1}{r}}{\frac{1}{r_1}-\frac{1}{r_2}}$$

and

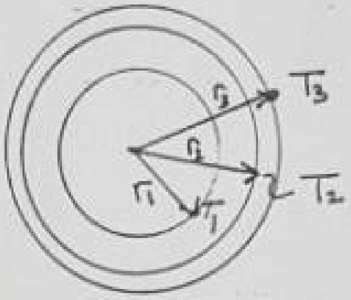
$$q = \frac{4\pi k (T_1 - T_2)}{\frac{1}{r_1}-\frac{1}{r_2}}$$

Thermal resistance



$$R = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

Composite spherical shell

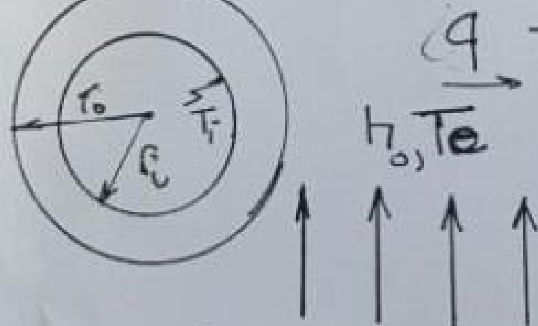


$$T_1 \circ \!-\!\!\!\!\!\underset{R_1}{\text{\textbackslash/\textbackslash/}}\!\!\!\!-\!\circ T_2 \!-\!\!\!\underset{R_2}{\text{\textbackslash/\textbackslash/}}\!\!\!-\!\circ T_3$$

$$R_1 = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

$$R_2 = \frac{r_3 - r_2}{4\pi k r_2 r_3}$$

Holbw Sphere, convection at the outer boundary surface



$$\xrightarrow{q} T_i \bullet\!\!-\!\!\underset{\frac{r_0 - r_i}{4\pi k r_0 r_i}}{\text{/\textbackslash/\textbackslash/}}\!\!-\!\!\bullet\!\!-\!\!\underset{\frac{1}{A_0 h_0}}{\text{/\textbackslash/\textbackslash/}}\!\!-\!\!\bullet T_e$$

$$q = \frac{(T_i - T_\infty)}{\dfrac{r_0 - r_i}{4\pi k r_0 r_i} + \dfrac{1}{4\pi r_0^2 h_0}}$$

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Conduction with Thermal Energy Generation

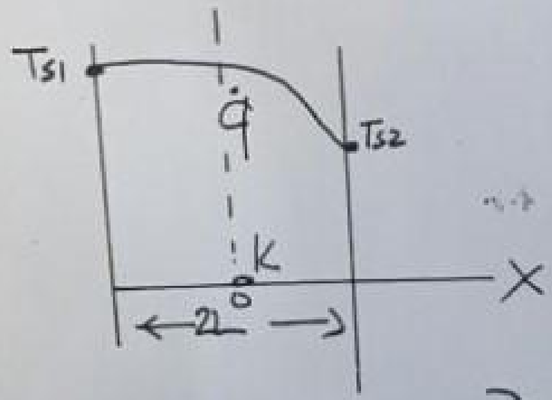
A common thermal energy generation involves conversion electrical energy to thermal energy in a current carrying medium (Joule heating)
Energy generated

$$\dot{E}_g = I^2 R \quad (W)$$

Volumetric generation rate (W/m^3)

$$\dot{q} = \frac{\dot{E}_g}{\forall} = \frac{I^2 R}{\forall} \quad (W/m^3)$$

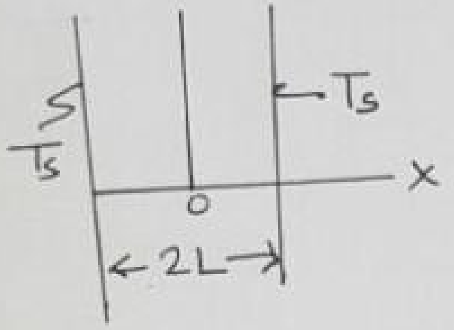
The Plane Wall



- Uniform energy generation

$$\left.\begin{aligned} &\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \\ &x = -L \quad T = T_{s1} \\ &x = L \quad T = T_{s2} \end{aligned}\right\} \quad T(x) = \frac{\dot{q}L^2}{2k}\left[1 - \frac{x^2}{L^2}\right] + \frac{T_{s2} - T_{s1}}{2}\frac{x}{L} + \frac{T_{s1} + T_{s2}}{2}$$

) $T_{s1} = T_{s2} = T_s$



$$\left.\begin{array}{ll} \dfrac{d^2T}{dx^2} + \dfrac{\dot{q}}{k} = 0 & \\ x = L & T = T_s \\ x = -L & T = T_s \\ \quad\text{or} & \\ x = 0 & \dfrac{dT}{dx} = 0 \end{array}\right\} \quad T(x) = \frac{\dot{q}L^2}{2k}\left[1 - \frac{x^2}{L^2}\right] + T_s$$

Maximum temperature occurs at $x=0$

$$T_0 = T(0) = T_s + \frac{\dot{q}L^2}{2k}$$

Dimensionless temperature

$$\frac{T(x) - T_s}{(\dot{q}L^2)/(2k)} = 1 - (x/L)^2$$

$$T_0 - T_s = \dot{q}L^2/(2k)$$

let $\eta = x/L$

$$\theta = \frac{T - T_s}{T_0 - T_s}$$

$$\theta(\eta) = 1 - \eta^2$$

see next page

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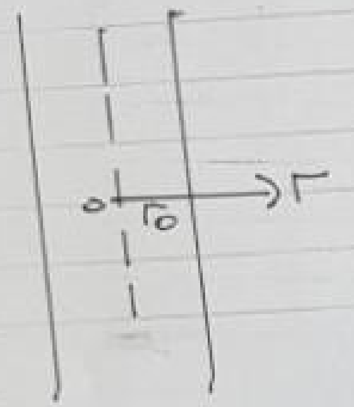
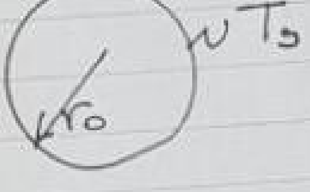
CYLINDER

a) Constant surface temperature





$$\dot{E}_g = I^2 R$$

$$\dot{q} = \frac{\dot{E}_g}{\forall} = \frac{I^2 R}{\forall}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

integrating

$$\frac{dT}{dr} = -\frac{\dot{q}r}{2k} + C_1/r$$

$$T(r) = -\frac{\dot{q}r^2}{4k} + C_1 \ln r + C_2$$

B.C.'s

$r = 0$ $\quad T = \text{const}$ or $\frac{dT}{dr} = 0$ (1)

$r = r_0$ $\quad T = T_s$ (2)

Recall

$$\left(\frac{dT}{dr}\right)_{r=0} = \frac{\dot{q}}{2k}\cdot 0 + \frac{C_1}{0} \Rightarrow C_1 = 0$$

$r = r_0$ $\quad T = T_s$ $\quad \therefore \quad T_s = C_2 + \left(-\frac{\dot{q}r_0^2}{4k}\right)$

$$C_2 = T_s + \frac{\dot{q}r_0^2}{4k}$$

$$T - T_s = \frac{\dot{q}r_0^2}{4k}\left[1 - r^2/r_0^2\right]$$

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Temperature at center of cylinder

$r=0 \quad T=T_c \quad \therefore \; T_c - T_s = \frac{\dot{q} r_o^2}{4k}$

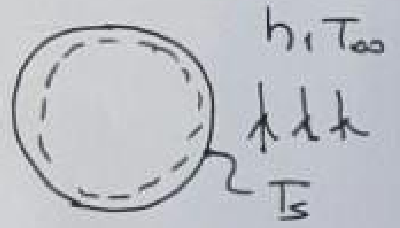
Heat transfer rate

$$q = -kA\left(\frac{dT}{dr}\right)_{r=r_o}$$

$$A = 2\pi r L$$

$$q = -k(2\pi r L)\left(-\frac{\dot{q} r}{2k}\right)\Big]_{r=r_o} = \pi r_o^2 L \dot{q}$$

b) convective boundary condition



$$\frac{1}{r}\frac{d}{dr}\left(k\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$r=0 \quad \frac{dT}{dr}=0$

$r=r_o \quad -k\left(\frac{dT}{dr}\right)_{r=r_o} = h\left[T - T_\infty\right]_{r=r_o}$

integrate

$$\frac{dT}{dr} = -\frac{\dot{q} r}{2k} + \frac{c_1}{r}$$

using B.C. $r=0 \quad \frac{dT}{dr}=0$:

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$C_1 = 0$

$\therefore$

$$\frac{dT}{dr} = -\frac{\dot{q}r}{2k}$$

integrate

$$T = -\frac{\dot{q}r^2}{4k} + C_2$$

Using B.C.

$$-k\left.\frac{dT}{dr}\right)_{r=r_o} = \left[h(T - T_\infty)\right]_{r=r_o}$$

$$C_2 = \frac{\dot{q}r_o^2}{4k} + \frac{\dot{q}r_o}{2h} + T_\infty$$

$$T(r) = T_\infty + \frac{\dot{q}}{4k}(r_o^2 - r^2) + \frac{\dot{q}r_o}{2h}$$

Maximum temperature occurs at center

$r = 0 \quad T = T_c$

$$T_c - T_\infty = \frac{\dot{q}r_o^2}{4k} + \frac{\dot{q}r_o}{2h}$$

in a different way writing energy balance

$$\dot{q}(\pi r_o^2 L) = h(2\pi r_o L)[T_s - T_\infty]$$