ME 313
Heat Transfer

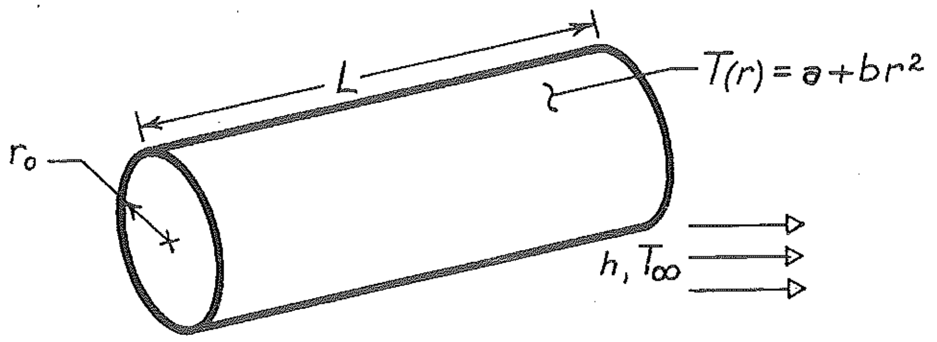
CHAPTER 2 EXAMPLES

1) A cylinder of radius r_0, length L, and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞. At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_0 and the fluid temperature.

KNOWN: Temperature distribution in solid cylinder and convection coefficient at cylinder surface.

FIND: Expressions for heat rate at cylinder surface and fluid temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: The heat rate from Fourier's law for the radial (cylindrical) system has the form

$$q_r = -kA_r \frac{dT}{dr}.$$

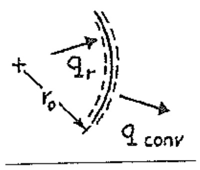
Substituting for the temperature distribution, $T(r) = a + br^2$,

$$q_r = -k(2\pi rL)\,2br = -4\pi kbLr^2.$$

At the outer surface ($r = r_o$), the conduction heat rate is

$$q_{r=r_o} = -4\pi kbLr_o^2. \quad <$$

From a surface energy balance at $r = r_o$,



$$q_{r=r_o} = q_{conv} = h(2\pi r_o L)\left[T(r_o) - T_\infty\right],$$

Substituting for $q_{r=r_o}$ and solving for T_∞,

$$T_\infty = T(r_o) + \frac{2kbr_o}{h}$$

$$T_\infty = a + br_o^2 + \frac{2kbr_o}{h}$$

$$T_\infty = a + br_o\left[r_o + \frac{2k}{h}\right]. \quad <$$

2) Consider an aluminum pan used to cook stew on top of an electric range. The bottom section of the pan is L = 0.25 cm thick and has a diameter of D = 18 cm. The electric heating unit on the range top consumes 900 W of power during cooking, and 90 percent of the heat generated in the heating elements transferred to the pan. During study steady operation, the temperature of the inner surface of the pan is measured to be 108 °C. Assuming temperature-dependent thermal conductivity and one dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem during steady operation. Do not solve.

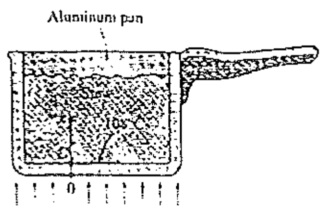


FIGURE P2-47

Heat conduction through the bottom section of an aluminum pan that is used to cook stew on top of an electric range is considered (Fig. P2-47). Assuming variable thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions 1. Heat transfer is given to be steady and one-dimensional. 2. Thermal conductivity is given to be variable. 3. There is no heat generation in the medium. 4. The top surface at $x = L$ is subjected to specified temperature and the bottom surface at $x = 0$ is subjected to uniform heat flux.

Analysis The heat flux at the bottom of the pan is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{G}}{\pi D^2/4} = \frac{0.90 \times (900\text{ W})}{\pi (0.18\text{ m})^2/4} = 31{,}831\text{ W/m}^2$$

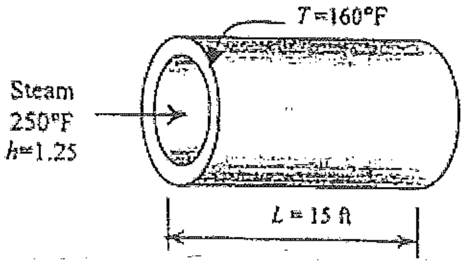
Then the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

$$-k\frac{dT(0)}{dr} = \dot{q}_s = 31{,}831\text{ W/m}^2$$

$$T(L) = T_L = 108°\text{C}$$

3) Consider a steam pipe of length L = 15 ft, inner radius r_1 = 2 in., outer radius r_2 = 2.4 in., and thermal conductivity k = 7.2 Btu/h.ft^2.^{0}F. Steam is flowing through the pipe at average temperature of 250^{0}F, and the average convection heat transfer coefficient on the inner surface is given to be h = 1.25 Btu/h.ft^2.^{0}F. If the average temperature on the outer surfaces of the pipe is T_2 = 160^{0}F, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (b) obtain a relation for the variation of temperature in the pipe by solving the differential equation, and (c) evaluate the rate of heat loss from the steam through the pipe.



solution

A steam pipe is subjected to convection on the inner surface and to specified temperature on the outer surface. The mathematical formulation, the variation of temperature in the pipe, and the rate of heat loss are to be determined for steady one-dimensional heat transfer.

Assumptions 1 Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. 2 Thermal conductivity is constant. 3 There is no heat generation in the pipe.

Properties The thermal conductivity is given to be $k = 7.2$ Btu/h·ft·°F.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and
$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$
$$T(r_2) = T_2 = 160°\text{F}$$

(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$
$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $\quad -k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)]$

$r = r_2$: $\quad T(r_2) = C_1 \ln r_2 + C_2 = T_2$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = \frac{T_2 - T_\infty}{\ln\frac{r_2}{r_1} + \frac{k}{hr_1}} \quad \text{and} \quad C_2 = T_2 - C_1 \ln r_2 = T_2 - \frac{T_2 - T_\infty}{\ln\frac{r_2}{r_1} + \frac{k}{hr_1}}\ln r_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_2 - C_1 \ln r_2 = C_1(\ln r - \ln r_2) + T_2 = \frac{T_2 - T_\infty}{\ln \frac{r_2}{r_1} + \frac{k}{hr_1}} \ln \frac{r}{r_2} + T_2$$

$$= \frac{(160-250)°\text{F}}{\ln \frac{2.4}{2} + \frac{7.2 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(12.5 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12 \text{ ft})}} \ln \frac{r}{2.4 \text{ in}} + 160°\text{F} = -24.74 \ln \frac{r}{2.4 \text{ in}} + 160°\text{F}$$

(*c*) The rate of heat conduction through the pipe is

$$\dot{Q} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi Lk \frac{T_2 - T_\infty}{\ln \frac{r_2}{r_1} + \frac{k}{hr_1}}$$

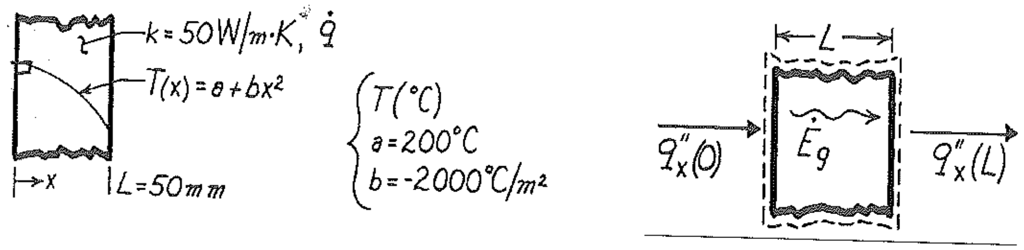
$$= -2\pi(15 \text{ ft})(7.2 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}) \frac{(160-250)°\text{F}}{\ln \frac{2.4}{2} + \frac{7.2 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(12.5 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12 \text{ ft})}} = \mathbf{16{,}800 \text{ Btu/h}}$$

4) The steady state temperature distribution in a one dimensional wall of thermal conductivity 50 W/mK and thickness 50 mm is observed to be $T(x) = a + bx^2$ where $a = 200\,°C$, $b = -2000\ °C/m^2$ and x in meters.

a) What is the heat generation rate $\dot{q}$ in wall?

b) determine heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.15 re-written as

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k\frac{d}{dx}\left[\frac{d}{dx}\left(a + bx^2\right)\right] = -k\frac{d}{dx}[2bx] = -2bk$$

$$\dot{q} = -2\left(-2000°C/m^2\right)\times 50\ W/m\cdot K = 2.0\times10^5\ W/m^3. \quad <$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k\left.\frac{dT}{dx}\right]_x .$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x) = -k\frac{d}{dx}\left[a + bx^2\right] = -2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0) = 0 \quad <$$

$$q''_x(L) = -2kbL = -2\times 50W/m\cdot K\left(-2000°C/m^2\right)\times 0.050m$$

$$q''_x(L) = 10{,}000\ W/m^2. \quad <$$

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(0) - q''_x(L) + \dot{q}L = 0$$

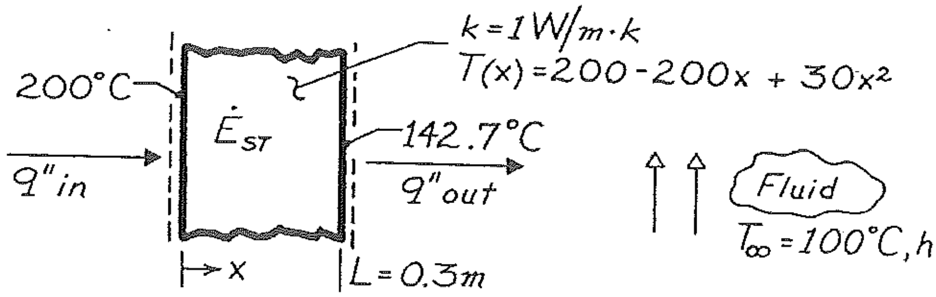
$$\dot{q} = \frac{q''_x(L) - q''_x(0)}{L} = \frac{10{,}000\ W/m^2 - 0}{0.050m} = 2.0\times10^5 W/m^3.$$

5) The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a+bx+cx^2$, where T is in degrees Celsius and x is in meters. a=200°C, b=-200 °C/m, and c=30 °C/m^2. The wall has a thermal conductivity of 1 W/mK.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(b) If the cold surface is exposed to a fluid at 100 °C, what is the convection coefficient?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant k.

ANALYSIS: (a) From Fourier's law,

$$q''_x = -k\frac{\partial T}{\partial x} = (200-60x)\cdot k$$

$$q''_{in} = q''_{x=0} = 200\frac{°C}{m}\times 1\frac{W}{m\cdot K} = 200\ W/m^2$$

$$q''_{out} = q''_{x=L} = (200-60\times 0.3)°\ C/m\ \times\ 1\ W/m\cdot K = 182\ W/m^2.$$

Applying an energy balance to a control volume about the wall, Eq. 1.11c,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$\dot{E}''_{st} = q''_{in} - q''_{out} = 18\ W/m^2.$$

(b) Applying a surface energy balance at x = L,

$$q''_{out} = h\left[T(L)-T_\infty\right]$$

$$h = \frac{q''_{out}}{T(L)-T_\infty} = \frac{182\ W/m^2}{(142.7-100)°\ C}$$

$$h = 4.3\ W/m^2\cdot K.$$

COMMENTS: (1) From the heat equation,

$$(\partial T/\partial t) = (k/\rho c_p)\ \partial^2 T/\partial x^2 = 60(k/\rho c_p),$$

it follows that the temperature is increasing with time at every point in the wall.

5) The temperature distribution in a plate of thickness 20 mm is given by

$$T(°C) = 10x + 6x^2 + 4.$$

Assume no heat generation in the plate, calculate heat flux on two sides of the plate. Also calculate the rate of temperature change with respect to time, if $k = 300$ W/mK, $\rho = 5800$ kg/m^3 and $c = 420$ J/kg K.

solution

$$T(x) = 6x^2 + 10x + 4$$

$L = 20$ mm $= 0.02$ m

$k = 300$ W/mK

$\rho = 5800$ kg/m^3

$c = 420$ J/kg K

$$\frac{k}{\rho c}\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)$$

$$\frac{\partial T}{\partial x} = 12x + 10$$

$$\frac{\partial^2 T}{\partial x^2} = 12$$

$$\therefore \frac{300}{(5800)(420)} \times \frac{\partial T}{\partial t} = 12$$

$$\frac{\partial T}{\partial t} = 97440 \text{ °C/s}$$

$\therefore$ rate of Temperature change with time

$$q_x = -k\left(\frac{dT}{dx}\right)_{x=0}$$

$$= -300\left[12x + 10\right]_{x=0}$$

$$= -3000 \text{ W/m}^2$$

$$q_{x=L} = -k\left(\frac{dT}{dx}\right)_{x=L}$$

$$= -300\left[12x + 10\right]_{x=0.02}$$

$$= -3072 \text{ W/m}^2$$