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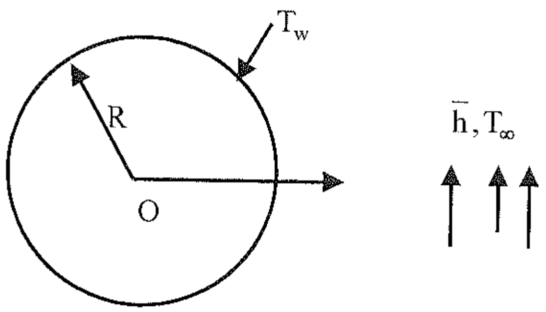
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT

ME 313 Heat Transfer
Quiz 3

FALL 2018

Consider a stainless-steel sphere [$k = 16\ W/m\ ^0C$] with uniform heat generation $\dot{q} = 1.0 MW/m^3$. Surface temperature of the sphere is $T_w = 464.4\ ^0C$ .Sphere has a dimeter of 4



cm and it is exposed to a convection environment at $T_\infty = 20\ ^0C$ with unknown heat transfer coefficient of $\bar{h}\ \left(W/m^2\ ^0C\right)$ .Assume steady-state one dimensional conduction in the sphere. [at any radius r the volume of the sphere is $\forall = \frac{4}{3}\pi r^3$ and surface area $A = 4\pi r^2$].

a) Develop the following differential equation that governs the temperature distribution in the sphere starting from basic principles (show all steps of your work)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

b) Write down the boundary conditions at $r = 0$ and at $r = r_0$

c) Show that the temperature distribution is given by

$$T = T_w + \left(\frac{\dot{q}}{6k}\right)\left(R^2 - r^2\right)$$

d) Calculate the center temperature of the sphere

e) Calculate the heat transfer coefficient by writing a surface energy balance on the sphere surface

Hint: $q'' = -k\frac{dT}{dr}$

$A q_r'' \rightarrow A q_r'' + \frac{d}{dr}(A q_r'')dr$

$$A q_r'' - \left[A q_r'' + \frac{d}{dr}(A q_r'')dr \right] + 4\pi r^2 \dot{q}\, dr = 0 \quad (2)$$

$$-\frac{d}{dr}\left[A q_r''\right] + 4\pi r^2 \dot{q} = 0$$

$$q_r'' = -K \frac{dT}{dr}$$

$$A = 4\pi r^2$$

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 \frac{dT}{dr}\right] + \frac{\dot{q}}{k} = 0$$

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = -\frac{\dot{q} r^2}{k}$$

$$\left(r^2 \frac{dT}{dr}\right) = -\frac{\dot{q} r^3}{3K} + C_1$$

$$\frac{dT}{dr} = -\frac{\dot{q} r}{3K} + \frac{C_1}{r^2}$$

Since $\frac{dT}{dr}(0) = 0 \rightarrow C_1 = 0$

$$\frac{dT}{dr} = -\frac{\dot{q} r}{3K}$$

$$T = -\frac{\dot{q} r^2}{6K} + C_2$$

$$r = R \quad T = T_w \longrightarrow C_2 = T_w + \frac{\dot{q} R^2}{6K}$$

$$T = T_w + \frac{\dot{q}}{6K}(R^2 - r^2)$$

d) $$T_c = T_w + \frac{\dot{q} R^2}{6K} = 464.4 + \frac{(10^6)\frac{W}{m^3}\cdot(0.02)^2}{6 \times 16}$$

$$= 468.6°C$$

e)

$$\frac{4}{3}\pi R^3 \dot{q} = 4\pi R^2 \bar{h}(T_w - T_\infty)$$

$$\frac{R}{3}\dot{q} = \bar{h}(T_w - T_\infty)$$

$$\bar{h} = 15 \ W/m^2 \, ^\circ C$$