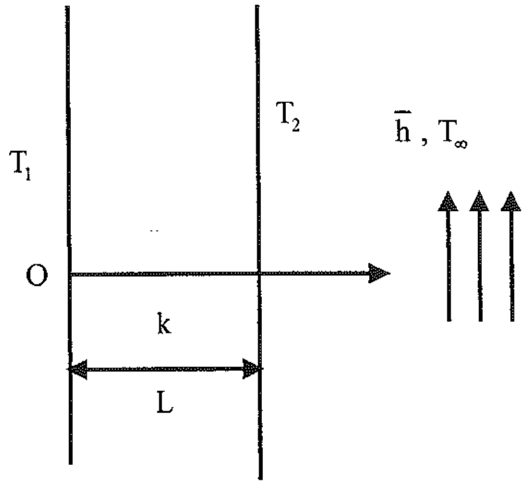
CANKAYA UNIVERSITY
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT

ME 313 Heat Transfer

FALL 2018
Quiz # 2

The temperature distribution across a AISI steel ($\rho = 8055\,kg/m^3, c_p = 480\,J/kg.K$, $k = 15.1\,W/m.K$) wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where T is in degrees Celsius and x is in meters, a= 200 ^{0}C, b=-200 ^{0}C/m, and c= 30 ^{0}C/m^2 .



a) Develop the equation that governs the temperature distribution (show all steps)

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

b) Determine wall temperature at x=0
c) Determine temperature at x=L

(c) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(d) If the cold surface is exposed to a fluid at 100 ^{0}C, what is the convective heat transfer coefficient $\bar{h}$?

(e) What is the rate of increase of temperature with time ?

a)

$A q_x'' \rightarrow \quad \rightarrow A q_x'' + \frac{\partial}{\partial x}(A q_x'')dx$

$A\,dx\,(\rho c_p)\frac{\partial T}{\partial t}$

$-\frac{\partial}{\partial x}(A q_x'') = \rho c_p \frac{\partial T}{\partial t}$

$$q''_k = -k\frac{\partial T}{\partial x}$$

$$k\frac{\partial^2 T}{\partial x^2} = \rho c_p \frac{\partial T}{\partial t}$$

b) $x = 0 \quad T = T_1 = a = 200°C$

c) $x = L \quad T = T_2 = (a + bL + cL^2)$

$$= 200 - 200 \times 0.3 + 30(0.3)^2$$

$$= 142.7°C$$

d) $q''_{x=0} = q''_{in} = -\left[k\frac{\partial T}{\partial x}\right]_{x=0} = -k(b + 2cx)\Big|_{x=0}$

$$= -k(-200 + 30 \times 2 \times 0)$$

$$= 200k = 200 \times 15.1 = 3020\ W/m^2$$

e) $q''_{x=L} = q''_{out} = -k\left(\frac{\partial T}{\partial x}\right)_{x=L} = -k\left[b + 2cx\right]_{x=L}$

$$= -k\left[-200 + 2 \times 30 \times 0.3\right] = 182k$$

$$= 2742.2\ W/m^2$$

Apply energy balance for stored energy $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$ ($\dot{E}_g = 0$)

$$\dot{E}_{st} = 3020 - 2742.2 = 271.8\ W/m^2$$

e) $\bar{h} = \frac{q''_{out}}{T(L) - T_\infty} = \frac{q''_{out}}{T_2 - T_\infty} = \frac{2742.2}{142.7 - 100} = 58.2\ \frac{W}{m^2K}$

f) $\frac{\partial T}{\partial t} = \frac{k}{(\rho c_p)}\frac{\partial^2 T}{\partial x^2} = \frac{k}{\rho c_p}[60]$

$$= \frac{15.1 \times 60}{(8055)(480)} = 0.000234\ °C/s$$