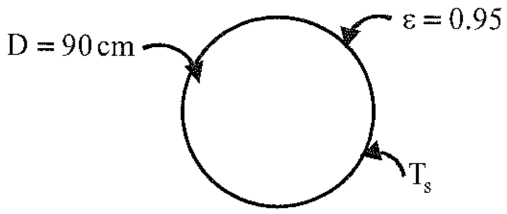
CANKAYA UNIVERSITY
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT

ME 313 Heat Transfer
Midter Exam I
FALL 2018
Show all steps of your work to get partial credit

1. The onboard power system of a small deep space probe steadily generates 3 kW energy. The probes' s components are enclosed by 90-cm diameter spherical aluminum shell. To minimize the surface temperature of the shell, and therefore the components within it, the shell's outside surface is covered by a very thin layer of special paint with an emissivity of $\varepsilon = 0.95$. What is the outside surface temperature of the aluminum shell? What is the heat flux?



outer space is at 0 K

1- steady state

$$\cancelto{0}{\dot{E}_{in}} - \dot{E}_{out} + \dot{E}_g = \cancelto{0}{\dot{E}_{st}}$$

$$\dot{E}_g = \dot{E}_{out}$$

$$= \epsilon\sigma A_s [T_s^4 - \cancelto{0}{T_{sur}^4}]$$

$$= \epsilon\sigma A_s T_s^4$$

$$T_s = \left[\frac{\dot{E}_g}{\epsilon\sigma A_s}\right]^{1/4} = \left[\frac{\dot{E}_g}{\epsilon\sigma\,\pi D^2}\right]^{1/4}$$

$$= \left[\frac{3000\ W}{(0.95)(0.9)^2(\pi)(5.669\times10^{-8})}\right]^{1/4} \cong 385\,K$$

2. Consider a plane wall of thickness L and thermal conductivity k. One side of the wall is exposed to a steady heat flux q'' , while the back side is maintained at temperature T_s

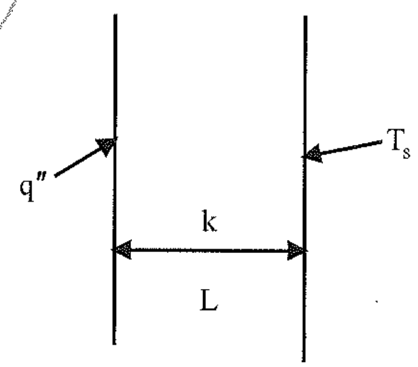
   a) Obtain the differential equation that governs the temperature distribution in the wall and write the boundary conditions

   b) Find the temperature distribution in the wall

   c) Find the temperature at x=0

   State your assumptions.

$$A q''_k \rightarrow \quad \rightarrow A q''_k + \frac{d}{dx}(A q''_k)\,dx$$



$$q''_k = -k\frac{dT}{dx}$$

so $\dfrac{d^2T}{dx^2} = 0$

$$-k\frac{dT}{dx}(0) = q''$$

$$T(L) = T_s$$

$$T = C_1 x + C_2$$

$$C_1 = -q''/k \qquad C_2 = \frac{q''}{k}L + T_s$$

$$\frac{T(x) - T_s}{\frac{q''L}{k}} = 1 - (x/L)$$

At $x = 0$ $\quad T(0) = \dfrac{q''L}{k} + T_s$

Assumption

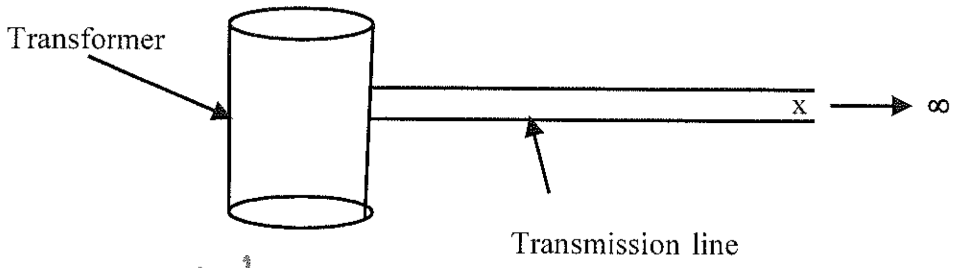
a) steady state

b) one dimensional conduction

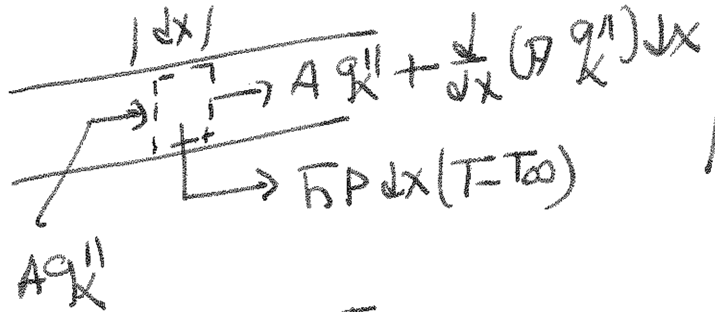
c) constant ρ, c, k

d) no energy generation

3) A transformer at a power station experiences a short circuit, resulting in a steady power dissipation of 300 W into a transmission line, which acts as a long fin emanating from the transformer. The transmission line has a diameter of 2 cm and is made of an aluminum alloy (k=190 W/m.K). The melting temperature of aluminum is 660 ^{0}C . The air temperature and heat transfer coefficient are 20 ^{0}C and $\bar{h} = 40\ \text{W}/\text{m}^2.\text{K}$ respectively. What is the temperature of the transmission line at its connection point with the transformer? Is the transmission line at risk of experiencing a failure?



1- Steady state
One dimensional conduction , $T = T(x)$
2- fin is infinitely long
3- no radiation
4- constant $\bar{h}$
5- constant properties



$$\frac{d^2T}{dx^2} - m^2 (T - T_\infty) = 0$$

$x = 0 \quad T = T_0$

$x = \infty \quad T = T_\infty$

$\theta = T - T_\infty$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad m = \left(\frac{\bar{h}P}{kA}\right)^{1/2}$$

$\theta(0) = \theta_0 \qquad \theta(\infty) = 0$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$C_1 = 0 \qquad C_2 = \theta_0$

$$\frac{\theta}{\theta_0} = \frac{T(x) - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

$$A = \frac{\pi D^2}{4} = 3.142 \times 10^{-4}\, m^2$$

$$P = \pi D = 0.0628\, m$$

$$q = -kA\left(\frac{d\theta}{dx}\right)_{x=0} = kAm\,\theta_0 = \theta_0\sqrt{\bar{h}PkA}$$

$$T_0 = \frac{q_0}{[\bar{h}PkA]^{1/2}} + T_\infty$$

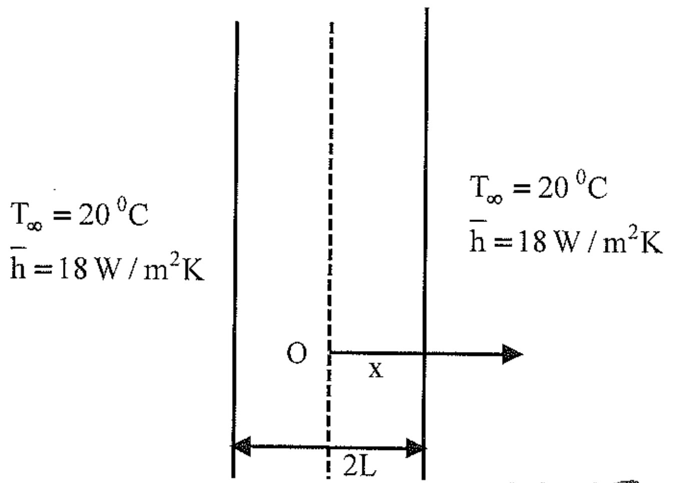
$$T_0 = \frac{390}{[40 \times 0.0628 \times 190 \times 3.142 \times 10^{-4}]^{1/2}} + 20^\circ C$$

$$= 795^\circ C$$

$T_0 > T_{melting}$

Fin will melt, there is a thermal failure.

4.) Upon removal from a $400\,^0C$ heat treating oven, a large plate of stainless steel ($\rho = 7900\,kg/m^3, c = 480\,J/kg.K, k = 15\,W/m.K$) is allowed to cool in $20\,^0C$, where the heat transfer coefficient is $18\,W/m^2.K$. If the plate is 25 cm thick, find the center and surface temperature of the plate after cooling for 2 hr.



$$Bi = \frac{\bar{h}L}{K} = \frac{(18\,W/m^2K)(0.125m)}{15\,W/mK} = 0.15 > 0.1$$

Use Heisler charts

$$\alpha = \frac{K}{\rho c_p} = \frac{15\,W/mK}{(7900\,kg/m^3)(480\,J/kg\,K)}$$

$$= 3.956 \times 10^{-6}\ m^2/s$$

$$Fo = \frac{\alpha t}{L^2} = \frac{(3.956 \times 10^{-6} \times 2\,hr \times \frac{3600s}{hr})}{(0.125m)^2} = 1.8229$$

$$\left.\begin{matrix} Fo = 1.8229 \\ 1/Bi = 6.66 \end{matrix}\right] \quad \frac{\theta_0}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} \sim 0.7891$$

$$T_0 = 0.7891(400-20)+20 = 320\,^\circ C$$

b) At $x = L$?

$$\frac{x}{L} = 1 \text{ and } \frac{1}{Bi} = 6.66 \Rightarrow \frac{\theta}{\theta_0} = 0.91$$

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} \simeq 0.91$$

$$T = T_\infty + 0.91\,(T_0 - T_\infty)$$

$$= 20 + 0.91(320 - 20)$$

$$= 293\,^\circ C$$