First Name-- **Date:November 21, 2015-**
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Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer
Midterm Exam I
Closed Notes Closed Book
Fall 2015
KEY
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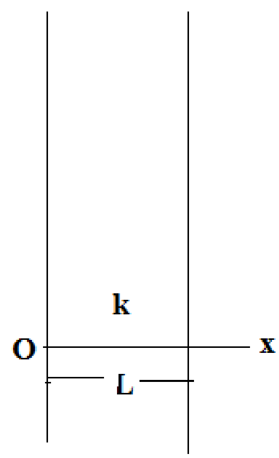
1) Steady-state temperature distribution in a one-dimensional wall of thermal conductivity 50 W/m.K and thickness 50 mm is observed to be

$$T(x) = a + bx^2$$

where $a = 200\,^0\mathrm{C}, b = -200\,^0\mathrm{C/m^2}$ and x is in meters and T is in $^0\mathrm{C}$.

(a) What is the heat generation rate $\dot{q}$ in the wall ?
(b) Determine heat fluxes at the two wall faces.

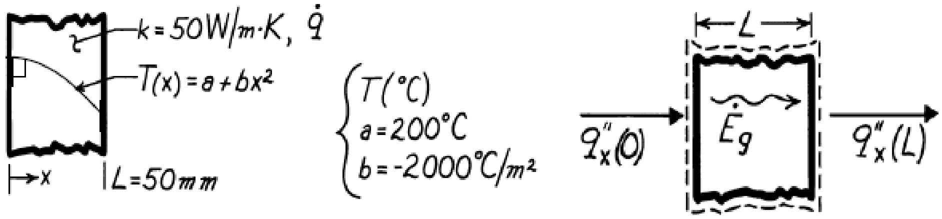


Solution

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.21 re-written as

$$\dot{q} = -k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right] = -k\frac{d}{dx}[2bx] = -2bk$$

$$\dot{q} = -2\left(-2000^\circ\text{C/m}^2\right)\times 50\ \text{W/m}\cdot\text{K} = 2.0\times 10^5\ \text{W/m}^3. \qquad <$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k\left.\frac{dT}{dx}\right]_x .$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x) = -k\frac{d}{dx}\left[a+bx^2\right] = -2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0) = 0 \qquad <$$

$$q''_x(L) = -2kbL = -2\times 50\text{W/m}\cdot\text{K}\left(-2000^\circ\text{C/m}^2\right)\times 0.050\text{m}$$

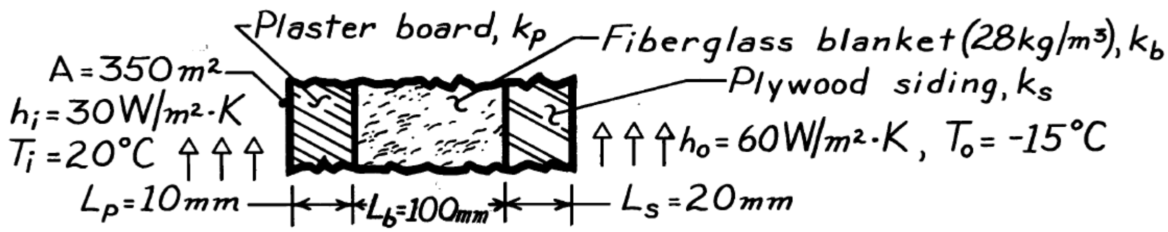
$$q''_x(L) = 10,000\ \text{W/m}^2. \qquad <$$

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q''_x(0) - q''_x(L) + \dot{q}L = 0$$

$$\dot{q} = \frac{q''_x(L) - q''_x(0)}{L} = \frac{10,000\ \text{W/m}^2 - 0}{0.050\text{m}} = 2.0\times 10^5\,\text{W/m}^3.$$

2)A house has a composite wall of wood, fiberglass insulation, and plaster board, as indicated in the sketch. On a cold winter day, the convection heat transfer coefficients are $h_0 = 60\,W/m^2.K$ and $h_i = 30\,W/m^2.K$. The total wall surface area is 350 m^2.



Determine the total heat flux through the wall.
Solution

PROPERTIES: *Table A-3*, $\left(\overline{T} = (T_i + T_o)/2 = (20-15)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m^3, k_b = 0.038 W/m·K; Plywood siding, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}. \qquad <$$

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{tot} = (T_i - T_o)/R_{tot}.$$

Substituting numerical values, find

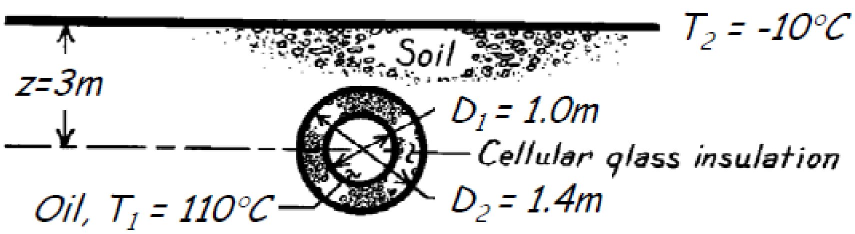
$$R_{tot} = \frac{1}{30W/m^2 \cdot K \times 350m^2} + \frac{0.01m}{0.17W/m \cdot K \times 350m^2} + \frac{0.10m}{0.038W/m \cdot K \times 350m^2} + \frac{0.02m}{0.12W/m \cdot K \times 350m^2} + \frac{1}{60W/m^2 \cdot K \times 350m^2}$$

$$R_{tot} = [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5}\ ^\circ C/W = 831 \times 10^{-5}\ ^\circ C/W$$

The heat loss is then,

$$q = \left[20-(-15)\right]^\circ C / 831 \times 10^{-5}\ ^\circ C/W = 4.21\ kW. \qquad <$$

3) A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 3 m below the surface. The pipe has an outer diameter of 1 m and is insulated with a layer of cellular glass 200 mm thick. What is the heat loss per unit length of pipe when heated oil at 110° C flows through the pipe and the surface of the earth is at a temperature of 10° C?



Solution

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through insulation, two-dimensional through soil, (3) Constant properties, (4) Negligible oil convection and pipe wall conduction resistances.

PROPERTIES: *Table A-3*, Soil (300K): k = 0.52 W/m·K; *Table A-3*, Cellular glass (365K): k = 0.069 W/m·K.

ANALYSIS: The heat rate can be expressed as

$$q = \frac{T_1 - T_2}{R_{tot}}$$

where the thermal resistance is $R_{tot} = R_{ins} + R_{soil}$. From Equation 3.33,

$$R_{ins} = \frac{\ell n(D_2/D_1)}{2\pi \ Lk_{ins}} = \frac{\ell n(1.4\text{m}/1\text{m})}{2\pi \ L \times 0.069 \ \text{W/m}\cdot\text{K}} = \frac{0.776\text{m}\cdot\text{K/W}}{L}.$$

From Equation 4.21 and Table 4.1,

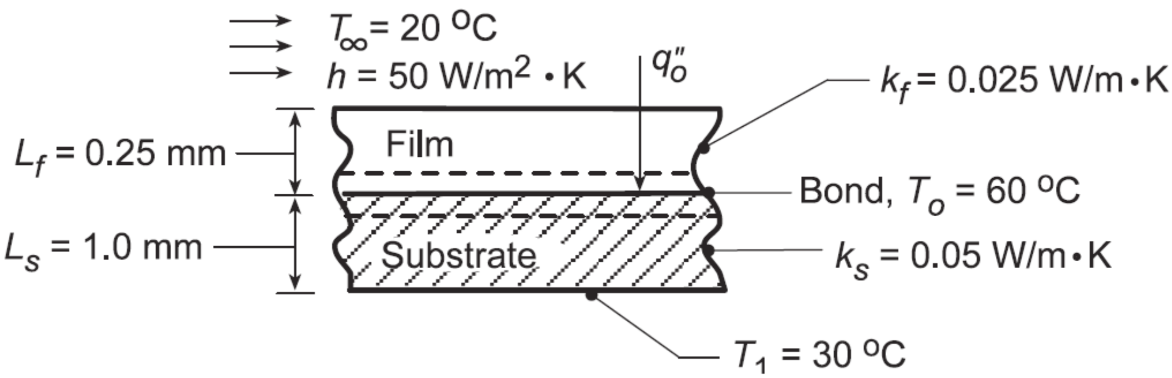
$$R_{soil} = \frac{1}{Sk_{soil}} = \frac{\cosh^{-1}(2z/D_2)}{2\pi \ Lk_{soil}} = \frac{\cosh^{-1}(6/1.4)}{2\pi \times (0.52 \ \text{W/m}\cdot\text{K})L} = \frac{0.653}{L}\text{m}\cdot\text{K/W}.$$

Hence,

$$q = \frac{(110-(-10))^{\circ}\text{C}}{\frac{1}{L}(0.776+0.653)\frac{\text{m}\cdot\text{K}}{\text{W}}} = 84\frac{\text{W}}{\text{m}} \times L$$

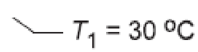
$$q' = q/L = 84 \ \text{W/m}. \qquad <$$

4) In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch.



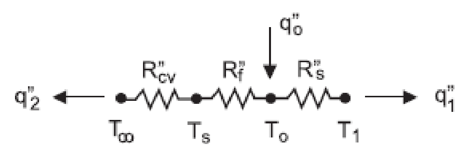
To cure the bond at a temperature T_0 , a radiant source is used to provide a heat flux $q_0''\,(W/m^2)$, all of which is absorbed at the bonded surface. The back of the substrate is maintained at T_1 while the free surface of the film is exposed to air at T_∞ and a convection heat transfer coefficient h. Calculate the heat flux $q_0''\,(W/m^2)$ required to maintain the bonded surface at $T_0 = 60\,^0C$.

Solution



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux q_o'' is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.



(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q_o'' = q_1'' + q_2'' \qquad\qquad q_o'' = \frac{T_o - T_\infty}{R_{cv}'' + R_f''} + \frac{T_o - T_1}{R_s''}$$

where the thermal resistances are

$$R_{cv}'' = 1/h = 1\big/50\,W\big/m^2\cdot K = 0.020\,m^2\cdot K/W$$

$$R_f'' = L_f/k_f = 0.00025\,m/0.025\,W/m\cdot K = 0.010\,m^2\cdot K/W$$

$$R_s'' = L_s/k_s = 0.001\,m/0.05\,W/m\cdot K = 0.020\,m^2\cdot K/W$$

$$q_o'' = \frac{(60-20)^\circ\,C}{[0.020+0.010]\,m^2\cdot K/W} + \frac{(60-30)^\circ\,C}{0.020\,m^2\cdot K/W} = (1333+1500)\,W\big/m^2 = 2833\,W\big/m^2 \quad <$$