First Name---

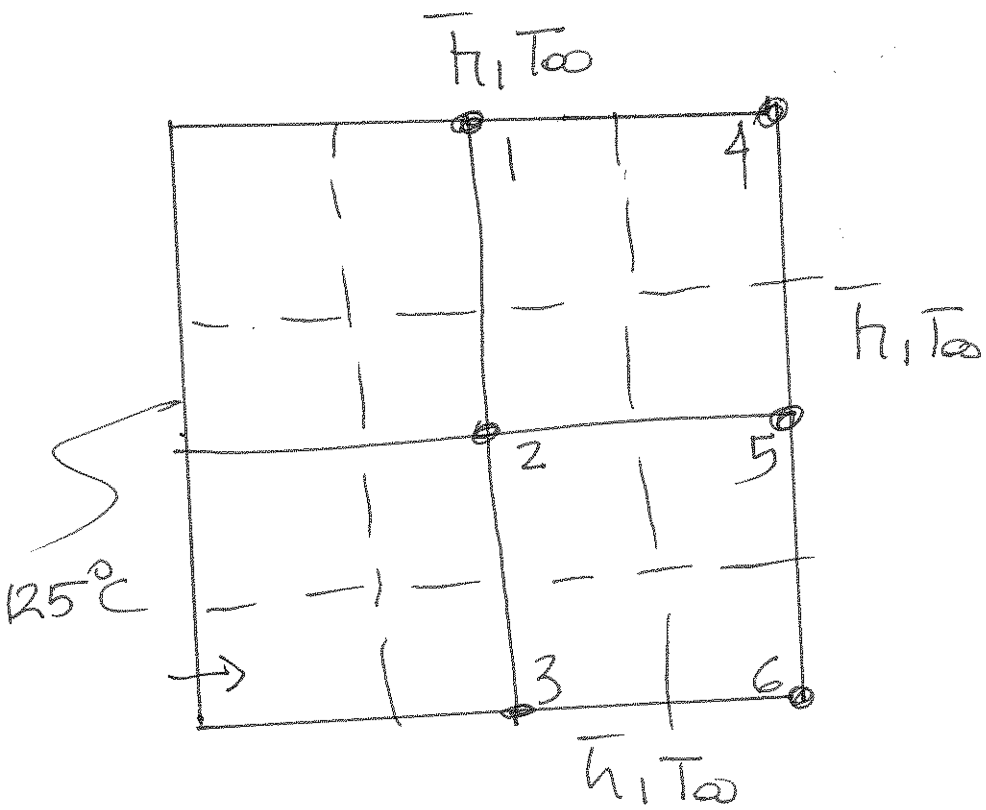
Last Name---

Student ID Number----------------------------

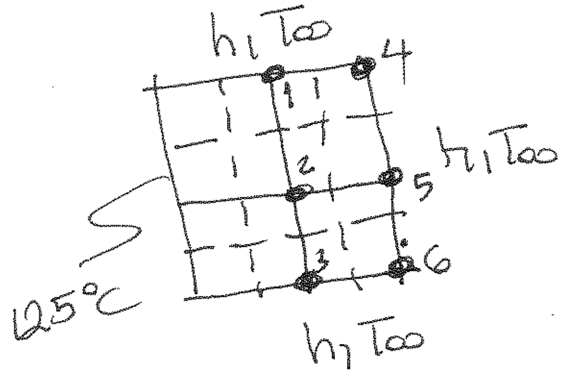
December 22, 2014-

Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer
Midterm Exam II
Closed Notes Open Book
Fall 2014

1) The square plate shown in figure has specified temperature of 125 ^{0}C on the left side. The other three sides are exposed to a 20 ^{0}C fluid where heat transfer coefficient is 30 W/m^2.K If the plate has a thermal conductivity of 6 W/m.K, Construct the algebraic equations determine two dimensional steady temperature distribution in the plate .Do not solve the equations.



2)



$T_\infty = 20°C$
$k = 6\ W/mK$
$h = 30\ W/m^2K$

Node-1

(1) $$\frac{k\Delta y}{2\Delta x}(125 - T_1) + h\Delta x(T_\infty - T_1) + \frac{k\Delta y}{2\Delta x}(T_4 - T_1) = 0$$

(2) $$\frac{k\Delta y}{\Delta x}(125 - T_2) + \frac{k\Delta x}{\Delta y}(T_1 - T_2) + \frac{k\Delta y}{\Delta x}(T_5 - T_2) + \frac{k\Delta x}{\Delta y}(T_3 - T_2) = 0$$

(3) $$\frac{k\Delta y/2}{\Delta x}(125 - T_3) + \frac{k\Delta x}{\Delta y}(T_2 - T_3) + \frac{k\Delta y/2}{\Delta x}(T_6 - T_3) + h\Delta x(T_\infty - T_6) = 0$$

(4) $$\frac{k\Delta y/2}{\Delta x}(T_1 - T_4) + \bar{h}\,\Delta x/2\,(T_\infty - T_4) + h\,\Delta y/2\,(T_\infty - T_4) + \frac{h\frac{\Delta x}{2}}{\Delta y}(T_5 - T_4) = 0$$

5) $$\frac{k\Delta x/2}{\Delta y}(T_4 - T_5) + \frac{k\Delta x}{\Delta y}(T_2 - T_5) + \frac{k\Delta x/2}{\Delta y}(T_6 - T_5) + \bar{h}\Delta y(T_\infty - T_5) = 0$$

6) $$\frac{k\Delta y/2}{\Delta x}(T_3 - T_6) + \frac{k\Delta x}{2\Delta y}(T_5 - T_6) + \frac{\bar{h}\Delta y}{2}(T_\infty - T_6) + \frac{\bar{h}\Delta x}{2}(T_\infty - T_6) = 0$$

2) Along cylindrical shaft of AISI 4130 steel with a radius of 4 cm has a uniform initial temperature of 100 ^{0}C .The shaft is then suddenly exposed to a convection environment in which the fluid temperature is 0 ^{0}C and heat transfer coefficient is 360 W/m^2.K. How long will it take for a point 8 mm beneath the shaft's surface to reach a temperature of 18 ^{0}C ? For AISI 1010 steel :$\rho = 7840\ kg/m^3$, $c_p = 460 \frac{J}{kg}.K$, $k = 43 \frac{W}{m}.K$ $\alpha = 1.19 \times 10^{-5}\ m^2/s$

$T_i = 100°C$ $\quad r_o = 0.04\,m$ $\quad \bar{h} = 360\ W/m^2K$

$T_\infty = 0°C$

$T = 18°C$ at 8 mm from surface

(a)

$$\left.\begin{aligned} \frac{r}{R} &= \frac{0.04-0.008}{0.04} = 0.8 \\ \frac{1}{Bi} &= \frac{k}{h r_o} = \frac{43}{(360)(0.04)} = 2.99 \end{aligned}\right\} \frac{\theta}{\theta_o} \cong 0.9$$

Use correction chart

$$\frac{\theta}{\theta_i} = \frac{T-T_\infty}{T_i-T_\infty} = \frac{18-0}{100-0} = 0.18$$

(b)

$$\frac{\theta}{\theta_i} = \frac{\theta_o}{\theta_i} \cdot \frac{\theta}{\theta_o}$$

$$0.18 = \left(\frac{\theta_o}{\theta_i}\right)(0.9)$$

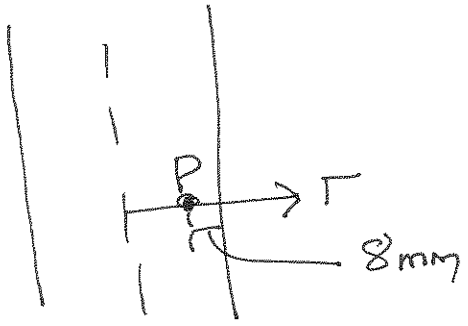
$$\frac{\theta_o}{\theta_i} = 0.18/0.9 = 0.20$$

(c)

now

$$\left.\begin{aligned} \frac{\theta_o}{\theta_i} &= 0.2 \\ 1/Bi &= 2.99 \end{aligned}\right\} F_o = 2.6$$

$$t = \frac{F_o r_o^2}{\alpha} = \frac{(2.6)(0.04)^2}{1.19 \times 10^{-5}} = 350\ s$$



$r_o = 40$ mm

$T_\infty = 0°C$

$h = 360$ W/m²°C

$T_i = 100°C$

$\frac{r}{r_o} = \frac{32}{40} = 0.8$

$$\theta^* = C_1 \exp(-\xi_1^2 F_o)\, J_0(\xi_1 r^*)$$

At $r = 40 - 8 = 32$ mm $\quad T = 18°C$

$$\frac{T - T_\infty}{T_i - T_\infty} = C_1 \exp[-\xi_1^2 F_o]\, J_0\left[\xi_1 \cdot (r/r_o)\right]$$

$$\underbrace{\frac{18-0}{100-0}}_{0.18} = C_1 \exp[-\xi_1^2 F_o]\, J_0(0.8\xi_1)$$

$$0.18 = C_1 \exp(-\xi_1^2 F_o)\, J_0(0.8\xi_1)$$

$$Bi = \frac{h r_o}{k} = \frac{(360)(4 \cdot 100)}{43} = 0.334 > 0.1$$

for $Bi = 0.334 \rightarrow \xi_1 \approx 0.8$

$C_1 \approx 1.082$

$$\exp(-\xi_1^2 F_o) = \frac{0.18}{C_1 J_0(0.8\xi_1)} = \frac{0.18}{(1.082)(\quad)}$$

$J_0(0.64) \approx 0.8966$

Table B4

$$\simeq 0.1855$$

$$-\xi_1^2 F_o = \ln(0.1855)$$

$$F_o = \frac{1.6847}{(0.8)^2} = 2.63$$

$F_o = \frac{\alpha t}{r_o^2} \; \rightarrow \; t \simeq 353$ s

3)Water at 10 ^{0}C flows at a velocity of 3 m/s across a plate maintained at a uniform temperature of 70 ^{0}C.If the length and width of the plate are 1.2 m and 1 m respectively, determine the convective heat transfer from the plate.

$T_\infty = 10\,°C$

$u_\infty = 3\,m/s$ $\quad T_s = 70\,°C$

$L = 1.2\,m \quad W = 1\,m$

Hint:
$\mu = \rho \nu$

$\nu = 65.64 \times 10^{-8}\,m^2/s$

$k = 0.63\,W/mK$

$Pr = 4.32$

$T_f = \frac{70+10}{2} = 40°C = 313\,K$

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(3)(1.2)}{65.64 \times 10^{-8}} = 5.48 \times 10^6 > 500\,000$$

turbulent flow

$$\overline{Nu_L} = [0.037 Re_L^{4/5} - 871] Pr^{1/3}$$

$$= [0.037(5.48 \times 10^6)^{0.8} - 871][4.32]^{1/3}$$

$$= 13\,410$$

$\nu = \frac{\mu}{\rho}$

$Re_L = \frac{u_\infty L}{\nu} = \frac{\rho u_\infty L}{\mu}$

$$\bar{h} = \frac{k}{L}(\overline{Nu_L}) = 7040\,W/m^2K$$

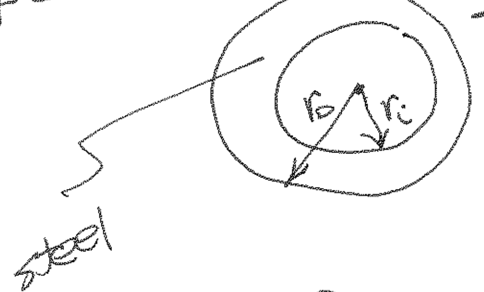
$$q = \bar{h} A (T_s - T_\infty) = 507\,kW$$

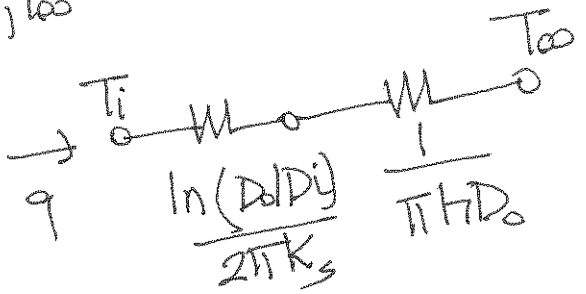
$$= (7040)(1.2)(1)(70-10)$$

4) Hot water at 50 ^{0}C is routed from one building in which it is generated to an adjoining building in which it is used for space heating. Transfer between the buildings occurs in a steelpipe (k =60 W/ m K) of 100- mm outside diameter and 8- mm wall thickness. During the winter, representative environmental conditions involve air at $T_\infty = -5\ ^0$C and V= 3 m/ s in cross flow over the pipe. What is the representative heat loss from an uninsulated pipe to the air per meter of pipe length? The convection heat transfer coefficient with water flow in the pipe is very large and for this reason we can assume that inside surface temperature of the pipe is 50 ^{0}C.

$\nu = 15.89 \times 10^{-6}\ m^2/s \qquad Pr = 0.707$

$T_f \cong 300K$





$Re_D = \frac{VD}{\nu} = 18880$

cross flow over cylinder

$$\overline{Nu}_D = 0.3 + \frac{0.62(18880)^{1/2}}{\left[1+\left(\frac{0.4}{0.707}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{18880}{282000}\right)^{5/8}\right]^{4/5}$$

$= 76.6$

$\bar{h} = \frac{k}{D}\overline{Nu}_D = 20.1\ W/m^2K$

$$\Sigma R = \frac{\ln(100/84)}{2\pi(60)} + \frac{1}{\pi(0.1)(20.1)} = 0.159$$

$$q = \frac{T - T_\infty}{\Sigma R} = \frac{50-(-5)}{0.159} = 346\ W/m$$