CANKAYA UNIVERSITY
FACULTY OF ENGINEERING
MECHANICAL ENGINEERING DEPARTMENT

ME 313 HEAT TRANSFER

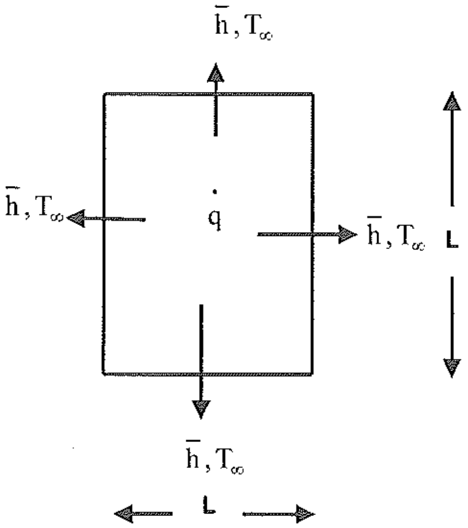
Fall 2016

HW on CHT

P 4.55 on text book

P 4.61 on text book

3) Consider a bar of square cross section.



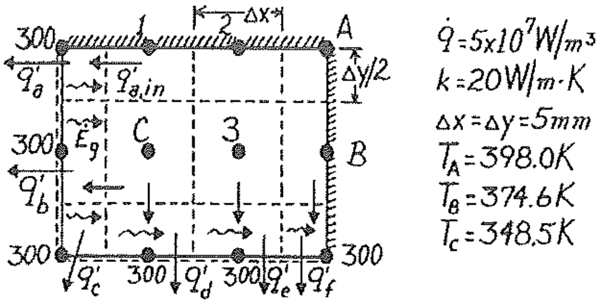
$\dot{q} = 10^6\ W/m^3, L = 20\,mm, k = 40\ W/m.K, \bar{h} = 700\ W/m^2K, T_\infty = 25\ {}^0C$

For this data determine the temperature distribution by choosing $\Delta x = \Delta y = 10\,mm$ grid.

KNOWN: Steady-state temperatures (K) at three nodes of a long rectangular bar.

FIND: (a) Temperatures at remaining nodes and (b) heat transfer per unit length from the bar using nodal temperatures; compare with result calculated using knowledge of $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, 2-D conduction. (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the nodes (1,2,3,A,B,C) can be written by inspection using Eq. 4.35 and recognizing that the adiabatic boundary can be represented by a symmetry plane.

$$\sum T_{neighbors} - 4T_i + \dot{q}\Delta x^2/k = 0 \quad \text{and} \quad \frac{\dot{q}\Delta x^2}{k} = \frac{5\times10^7\ \text{W/m}^3(0.005\text{m})^2}{20\ \text{W/m}\cdot\text{K}} = 62.5\text{K}.$$

Node A (to find T_2*):*

$$2T_2 + 2T_B - 4T_A + \dot{q}\Delta x^2/k = 0$$

$$T_2 = \frac{1}{2}(-2\times374.6 + 4\times398.0 - 62.5)\text{K} = 390.2\text{K} \qquad <$$

Node 3 (to find T_3*):*

$$T_c + T_2 + T_B + 300\text{K} - 4T_3 + \dot{q}\Delta x^2/k = 0$$

$$T_3 = \frac{1}{4}(348.5 + 390.2 + 374.6 + 300 + 62.5)\text{K} = 369.0\text{K} \qquad <$$

Node 1 (to find T_1*):*

$$300 + 2T_C + T_2 - 4T_1 + \dot{q}\Delta x^2/k = 0$$

$$T_1 = \frac{1}{4}(300 + 2\times348.5 + 390.2 + 62.5) = 362.4\text{K} \qquad <$$

(b) The heat rate out of the bar is determined by calculating the heat rate out of each control volume around the 300 K nodes. Consider the node in the upper left-hand corner; from an energy balance

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \quad \text{or} \quad q'_a = q'_{a,in} + \dot{E}_g \quad \text{where} \quad \dot{E}_g = \dot{q}V.$$

Hence, for the entire bar $\quad q'_{bar} = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$, or

$$q'_{bar} = \left[k\frac{\Delta y}{2}\frac{T_1 - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_a + \left[k\Delta y\frac{T_C - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\Delta y\right]\right]_b + \left[\dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_c +$$

$$\left[k\Delta x\frac{T_C - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_d + \left[k\Delta x\frac{T_3 - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_e + \left[k\frac{\Delta x}{2}\frac{T_B - 300}{\Delta y} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_f.$$

Substituting numerical values, find $q'_{bar} = 7,502.5$ W/m. From an overall energy balance on the bar,

$$q'_{bar} = \dot{E}'_g = \dot{q}V/\ell = \dot{q}(3\Delta x\cdot2\Delta y) = 5\times10^7\ \text{W/m}^3\times6(0.005\text{m})^2 = 7,500\ \text{W/m}. \qquad <$$

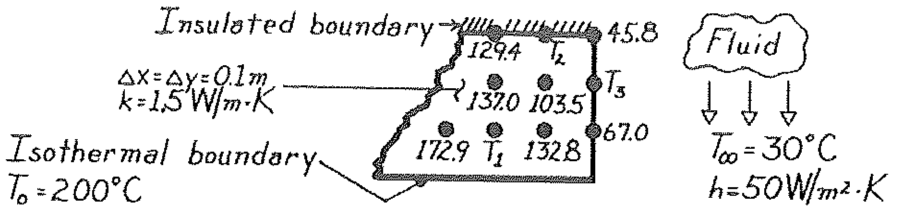
As expected, the results of the two methods agree. Why must that be?

P4.61

KNOWN: Steady-state temperatures (°C) associated with selected nodal points in a two-dimensional system.

FIND: (a) Temperatures at nodes 1, 2 and 3. (b) Heat transfer rate per unit thickness from the system surface to the fluid.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction. (2) Constant properties.

ANALYSIS: (a) Using the finite-difference equations for Nodes 1, 2 and 3:

Node 1, Interior node, Eq. 4.29: $T_1 = \frac{1}{4} \cdot \sum T_{neighbors}$

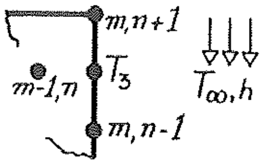
$$T_1 = \frac{1}{4}(172.9 + 137.0 + 132.8 + 200.0)^\circ C = 160.7^\circ C \qquad <$$

Node 2, Insulated boundary, Eq. 4.46 with h = 0, $T_{m,n} = T_2$

$$T_2 = \frac{1}{4}\left(T_{m-1,n} + T_{m+1,n} + 2T_{m,n-1}\right)$$

$$T_2 = \frac{1}{4}(129.4 + 45.8 + 2 \times 103.5)^\circ C = 95.6^\circ C \qquad <$$

Node 3, Plane surface with convection, Eq. 4.42, $T_{m,n} = T_3$



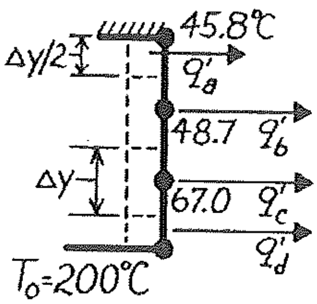
$$2\left[\frac{h\Delta x}{k} + 2\right]T_3 = \left(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}\right) + \frac{2h\Delta x}{k}T_\infty$$

$$h\Delta x/k = 50 W/m^2 \cdot K \times 0.1m/1.5W/m \cdot K = 3.33$$

$$2(3.33 + 2)T_3 = (2 \times 103.5 + 45.8 + 67.0)^\circ C + 2 \times 3.33 \times 30^\circ C$$

$$T_3 = \frac{1}{10.66}(319.80 + 199.80)^\circ C = 48.7^\circ C \qquad <$$

(b) The heat rate per unit thickness from the surface to the fluid is determined from the sum of the convection rates from each control volume surface.

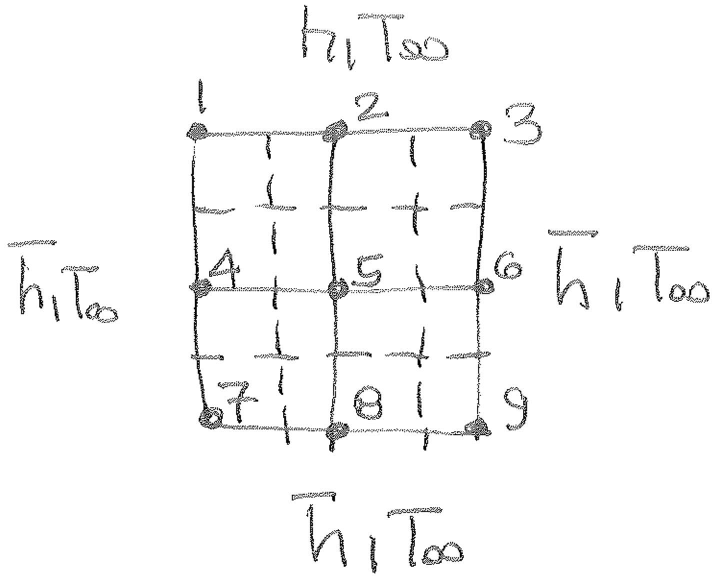


$$q'_{conv} = q'_a + q'_b + q'_c + q'_d$$

$$q_i = h\Delta y_i (T_i - T_\infty)$$

$$q'_{conv} = 50\frac{W}{m^2 \cdot K}\left[\frac{0.1}{2}m\ (45.8 - 30.0)^\circ C + 0.1m\ (48.7 - 30.0)^\circ C + 0.1m\ (67.0 - 30.0)^\circ C + \frac{0.1m}{2}(200.0 - 30.0)^\circ C\right]$$

$$q'_{conv} = (39.5 + 93.5 + 185.0 + 425)\ W/m = 743\ W/m. \qquad <$$



$\Delta x = 10$ mm
$\Delta y = 10$ mm

(1)
$$(\bar h \Delta x/2)(T_\infty - T_1) + \frac{h\Delta y}{2}(T_\infty - T_1) + \frac{k\Delta y}{2\Delta x}(T_2 - T_1) + \frac{k\Delta x}{2\Delta y}(T_4 - T_1) + \dot q\left(\frac{\Delta x}{2}\right)\left(\frac{\Delta y}{2}\right) = 0$$

(2)
$$\bar h \Delta x(T_\infty - T_2) + \frac{k\Delta y}{2\Delta x}(T_3 - T_2) + \frac{k\Delta x}{\Delta y}(T_5 - T_2) + \frac{k\Delta y}{2\Delta x}(T_1 - T_2) + \dot q(\Delta y/2)(\Delta x) = 0 \quad +$$

(3)
$$\bar h \frac{\Delta x}{2}(T_\infty - T_3) + \bar h\frac{\Delta y}{2}(T_\infty - T_3) + \frac{k\Delta x}{2\Delta y}(T_6 - T_3) + \frac{k\Delta y}{2\Delta x}(T_2 - T_3) + \dot q \frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

4)
$$\bar h \Delta y(T_\infty - T_4) + \frac{k\Delta x}{2\Delta y}(T_1 - T_4) + \frac{k\Delta x}{\Delta y}(T_5 - T_4) + \frac{k\Delta x}{2\Delta y}(T_7 - T_4) + \dot q \Delta y \frac{\Delta x}{2} = 0$$

5) $$\frac{k\Delta x}{\Delta y}(T_2-T_5)+\frac{k\Delta y}{\Delta x}(T_6-T_5)+\frac{k\Delta x}{\Delta y}(T_8-T_5)$$ 5)

$$+\frac{k\Delta y}{\Delta x}(T_4-T_5)+\dot{q}\,\Delta x\Delta y=0$$

6) $$\frac{k\Delta y}{\Delta x}(T_5-T_6)+\frac{k\Delta x}{2\Delta y}(T_3-T_6)+\bar{h}\Delta y(T_\infty-T_6)$$

$$+\frac{k\Delta x}{2\Delta y}(T_9-T_6)+\dot{q}\,\Delta y\frac{\Delta x}{2}=0$$

7) $$\bar{h}\frac{\Delta x}{2}(T_\infty-T_7)+\frac{k\Delta x}{2\Delta y}(T_4-T_7)+\frac{k\Delta y}{2\Delta x}(T_\infty-T_7)$$

$$+\bar{h}\frac{\Delta x}{2}(T_\infty-T_7)+\dot{q}\,\frac{\Delta x}{2}\frac{\Delta y}{2}=0$$

8) $$\frac{k\Delta y}{2\Delta x}(T_7-T_8)+\frac{k\Delta x}{\Delta y}(T_5-T_8)+\frac{k\Delta y}{2\Delta x}(T_9-T_\infty)$$

$$+\bar{h}\Delta x(T_\infty-T_\infty)+\dot{q}\,\Delta x\frac{\Delta y}{2}=0$$

9) $$\frac{k\Delta y}{2\Delta x}(T_\infty-T_9)+\frac{k\Delta x}{2\Delta y}(T_6-T_9)+\frac{\bar{h}\Delta y}{2}(T_\infty-T_9)$$

$$+\bar{h}\frac{\Delta x}{2}(T_\infty-T_9)+\dot{q}\left(\frac{\Delta x}{2}\frac{\Delta y}{2}\right)=0$$