CANKAYA UNIVERSITY
FACULTY OF ENGINEERING
MECHANICAL ENGINEERING DEPARTMENT
ME 313 HEAT TRANSFER

Fall 2016

HW 5 SOLUTIONS

1) A solid copper sphere of 10- cm diameter initially at a uniform temperature $T_i = 250\,^0C$, is suddenly immersed in a well stirred fluid which is maintained at a uniform temperature $T_\infty = 50\,^0C$. The heat transfer coefficient between the sphere and the fluid is $\bar{h} = 200\ W/m^2\,^0C$ .Determine the temperature of the copper block for t=5 sec.$\left[\rho = 8954\,kg/m^3, c_p = 383\,J/kg.^0C, k = 386\,W/m\,^0C\right]$.

$$L_c = \frac{\forall}{A} = \frac{\frac{4}{3}\pi r_0^3}{4\pi r_0^2} = \frac{r_0}{3} = \frac{D}{6} = \frac{0.1m}{6} = 0.0166\ m$$

$$Bi = \frac{\bar{h}L_c}{k} = \frac{(200)(0.0166)}{386} = 8.6\times10^{-3} < 0.1$$

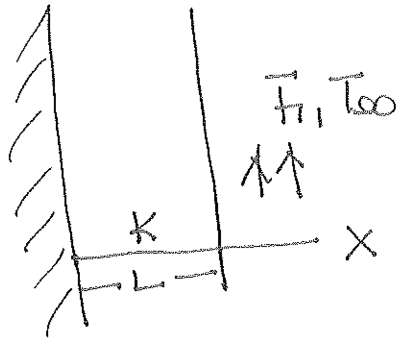
Lumped parameter

$$\frac{T-T_\infty}{T_i-T_\infty} = \exp(-mt)$$

$$m = \frac{\bar{h}A}{\rho c_p \forall} = 3.5\times10^{-3}\ 1/s$$

$$T = T_\infty + (T_i - T_\infty)e^{-mt} = 246.53\ ^oC$$

2) P5.51 of textbook



$L = 0.15\,m$

$T_i = 20°C$

$T_\infty = 950°C$

$T_o = 750°C$

$\bar{h} = 100\ W/m^2K$

$\rho = 2600\ kg/m^3$

$c_p = 1000\ J/kgK$

$k = 1.5\ W/mK$

1) one dimensional transient conduction
2) ρ, c_p, k constant
3) adiabatic outer surface
4) $Fo > 0.2$
5) no radiation from combustion gases

$$\theta_o^* = \frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = 0.215$$

$$\frac{1}{Bi} = \frac{k}{\bar{h}L} = 0.1$$

$Fo \simeq 0.867$

$$t = \frac{Fo\,L^2}{\alpha} = 33\,800\ sec$$

Method 2

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = 0.215$$

$$Bi = \frac{\bar{h}L}{k} = 10$$

$$Fo = \alpha t / L^2 \geq 0.2$$

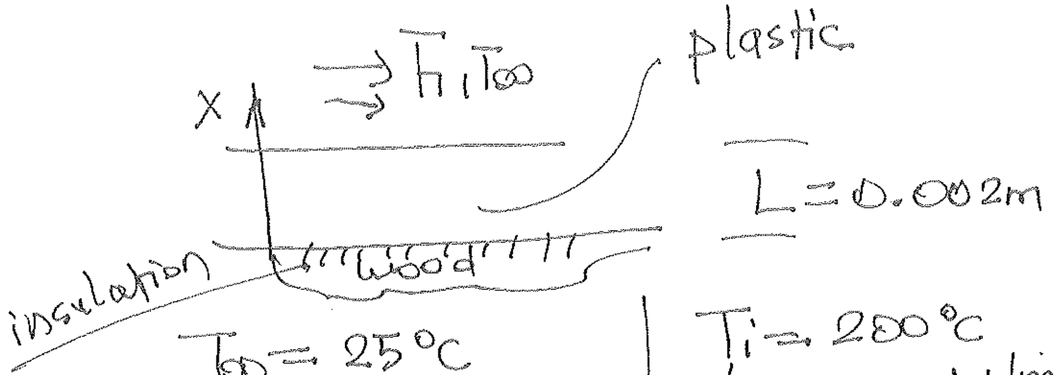
$\xi_1 = 1.4289$
$C_1 = 1.262$) table 5.1

$$Fo = \frac{\ln(\theta_o^*/C_1)}{\xi_1^2} = 0.867$$

$$t = \frac{Fo\,L^2}{\alpha} = 33\,800\ s$$

(2/5)

3) P5.59 of textbook



$T_\infty = 25\,^\circ C$ | $T_i = 200\,^\circ C$

$\bar{h} = 200\ W/m^2K$ | $k = 0.25\ W/mK$

$\alpha = 1.2\times10^{-7}\ m^2/s$

1) one dimensional transient conduction
2) ρ, c, k const.
3) no heat transfer at interface

$$Bi = \frac{\bar{h}L}{k} = \frac{(0.002\,m)(200)}{0.25} = 1.6 > 0.1$$ use charts

From Table 5.1: $C_1 = 1.155$

$\xi_1 = 0.99$

Surface temperature ratio

$$\theta_s^* = \frac{T_s - T_\infty}{T_i - T_\infty} = \frac{42-25}{200-25} = 0.0971$$

$$\theta^* = C_1 \exp(-\xi_1^2 Fo)\cos(\xi_1 x^*)$$

$$x^* = \frac{x}{L} = 1$$

$$\theta_s^* = C_1 \exp(-\xi_1^2 Fo)\cos(\xi_1)$$

$$\exp(-\xi_1^2 F_o) = \frac{\theta_s^*}{C_1 \cos(\xi_1)}$$

$$F_o = \frac{-\ln\left[\frac{\theta_s^*}{C_1 \cos(\xi_1)}\right]}{\xi_1^2} = 1.914$$

$$t = \frac{F_o L^2}{\alpha} = 63.8 \text{ s}$$

Method 2

Position correction chart

$$\frac{X}{L} = 1$$

$$\frac{K}{hL} = \frac{1}{1.6} = 0.625$$

$$\frac{\theta}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} \cong 0.65$$

$$T_o = T_\infty + \frac{(T - T_\infty)}{0.65}$$

$$\approx 25 + \frac{42 - 25}{0.65} \cong 51\ {}^\circ C$$

Midplane chart

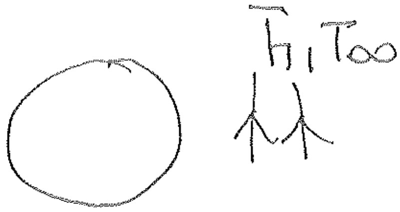
$$\frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{51 - 25}{200 - 25} \cong 0.148$$

$$\frac{1}{Bi} = 0.625$$

$$F_o \cong 1.92$$

$$t = \frac{F_o L^2}{\alpha} \approx 64 \text{ s}$$

4) P5.61 of textbook



$T_i = 1000\,K$

$\rho = 400\ kg/m^3$

$c = 1600\ J/kg\,K$

$k = 1.7\ W/m\,K$

$\bar{h} = 50\ W/m\,K$

$T_\infty = 350\,K$

$D = 30\ mm$

$$Bi = \frac{\bar{h} L_c}{K} = \frac{\bar{h}[r_o/2]}{K} = 0.221 > 0.1$$

charts will be used

$$\theta^* = C_1 \exp[-\xi_1^2 F_o]\, J_o(\xi_1 r^*)$$

At surface $r^* = r/r_o = 1$

$$\theta_s^* = C_1 \exp[-\xi_1^2 F_o]\, J_o(\xi_1)$$

$$F_o = -\frac{1}{\xi_1^2} \ln\left[\frac{\theta_s^*}{C_1 J_o(\xi_1)}\right]$$

$$\theta_s^* = \frac{T_s - T_\infty}{T_i - T_\infty} = \frac{500 - 350}{1000 - 350} = 0.231$$

$$Bi = \frac{h r_o}{K} = 0.441 \longrightarrow \left.\begin{array}{l} \xi_1 = 0.8882 \\ C_1 = 1.1019 \end{array}\right\} \text{from table}$$

$\therefore F_o = 1.72$

$$t = F_o r_o^2/\alpha = F_o r_o^2 \frac{\rho c}{K} = 145\ s$$

(5/5)

Method 2

Position correction chart

$$\left.\begin{array}{l} \frac{r}{r_0} = 1 \\ \frac{1}{Bi} = \frac{k}{h r_0} = 2.26 \end{array}\right] \quad \frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} \cong 0.81$$

$$\therefore T_0 = T_\infty + \frac{T - T_\infty}{0.81} = 350 + \frac{500 - 350}{0.81}$$

$$\cong 535\ K$$

~~Mid~~ center temperature

$$\left.\begin{array}{l} \frac{\theta_0}{\theta_i} = \frac{535 - 350}{1000 - 350} \cong 0.28 \\ \frac{1}{Bi} = 2.26 \end{array}\right] F_0 \approx 1.70$$

$$t = \frac{F_0\, r_0^2}{\alpha} \cong 145\ s$$

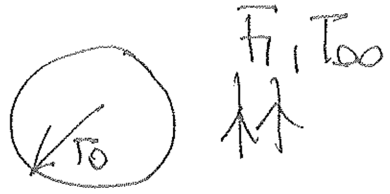
5) P5.71 of textbook

$\rho = 7800\ kg/m^3$

$c = 500\ J/kg\,K$

$k = 50\ W/m\,K$

$D = 50\ mm$



$T_\infty = 1300\ K \qquad \bar{h} = 4000\ W/m^2 K$

$T_i = 300\ K \qquad r = 12.5 - 1 = 11.5\ mm$

$Bi = \frac{\bar{h} L_c}{K} = \frac{\bar{h}(r_o/3)}{K} = 0.33 > 0.1$ use charts

$Bi = \frac{h r_o}{K} = 1$

$$F_o = -\frac{1}{\zeta_1^2} \ln \left[\frac{\theta^*}{\frac{C_1}{\zeta_1 r^*} \sin(\zeta_1 r^*)} \right]$$

$Bi = 1 \Longrightarrow$ From Table $\zeta_1 = 1.5708 \quad C_1 = 1.2732$

$r^* = \frac{r}{r_o} = \frac{11.5}{12.5} = 0.92$

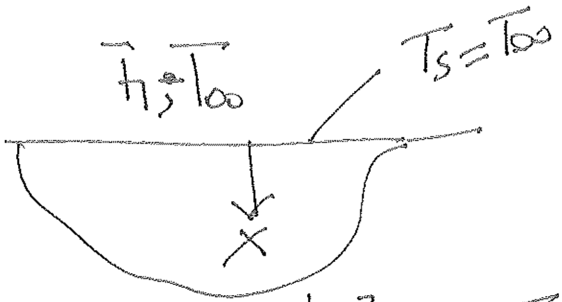
$$F_o = \frac{-1}{(1.5702)^2} \ln \left[\frac{\frac{(1000-1300)}{(300-1300)}}{\frac{1.2732}{(1.5708)(0.92)} \sin(1.5708 \times 0.92)} \right]$$

$= 0.433$

$t = F_o \frac{r_o^2}{\alpha} = F_o\, r_o^2\, \rho \frac{c}{k} = 5.3\ s$

(7/5)

6) P5.86 of textbook



$T_\infty = 30°C$

$\rho = 7800 \ kg/m^3$
$c = 480 \ J/kg\,K$
$k = 50 \ W/mK$

$T_i = 300°C$
$T_\infty = 30°C$

$T = 50°C$

At $x = 50 \ mm$

Semi infinite body

$$\frac{T - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\frac{T - T_\infty}{T_i - T_s} = \frac{50 - 30}{300 - 30} = 0.0741$$

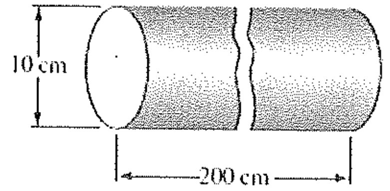
$$\left(\frac{x}{2\sqrt{\alpha t}}\right) \cong 0.0657$$ for 0.0741 from table of error function

$$\frac{x^2}{4\alpha t} = (0.0657)^2$$

$$t = \frac{(0.02)^2}{(0.0261)(1.34 \times 10^{-5})} = 1730 \ s$$

(8|5)

7) A stainless steel cylindrical billet (k _=14.4 W/m K, $\alpha = 3.9\times10^{-6}\ m^2/s$) is heated to 593°C preparatory to a forming process. If the minimum temperature permissible for forming is 482°C, how long can the billet be exposed to air at 38°C if the average heat transfer coefficient is 85 W/m2 K? The shape of the billet is shown in the sketch.



$T_i = 393\ °C$
$T_s = 482\ °C$ at $r = r_o$
$T_\infty = 38\ °C$

$Bi = \frac{h r_i}{k} = ?$

$$L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{r_o^2 L}{2 r_o L + 2 r_o^2} = \frac{r_o L}{2(r_o L + r_o)}$$

$= 0.02439$

$Bi = \frac{h L_c}{k} = 0.143 > 0.1$ use charts

infinite cylinder; $L >> r_o$

use position correction chart

$$\left.\begin{aligned} \frac{r}{r_o} &= 1 \\ \frac{1}{Bi} &= \frac{k}{h r_o} = 3.33 \end{aligned}\right] \quad \frac{\overbrace{T}^{482\,°C} - T_\infty}{T_o - T_\infty} = 0.87 \rightarrow T_o = 548\ °C$$

surface temperature 482 °C

center temperature chart

$$\left[\begin{aligned} \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{548-38}{593-38} = \frac{510}{555} \cong 0.92 \\ \frac{1}{Bi} &= \frac{k}{h r_o} = 3.33 \end{aligned}\right] \quad F_o \cong 0.2$$

$$t = \frac{F_o r_o^2}{\alpha} = \frac{(0.2)(0.05)^2}{3.9\times10^{-6}} \cong 130\ s$$

(9/5)