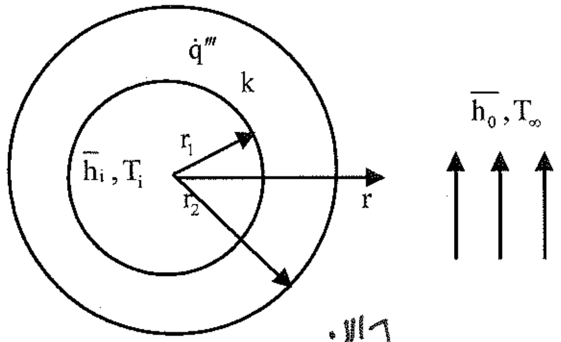
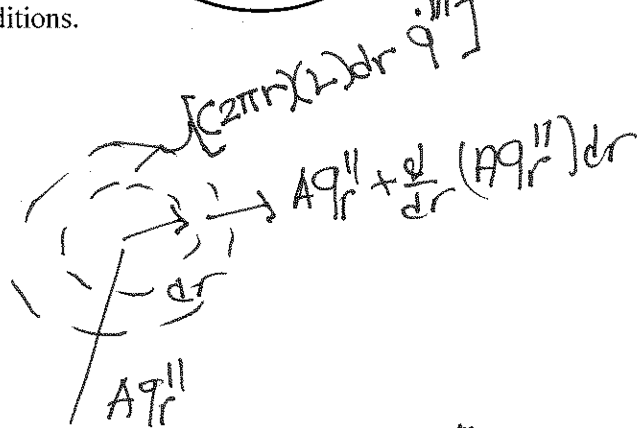
Cankaya University
Faculty of Engineering
Mechanical Engineering
Chapter 2 Examples
Fall 2018

1) Consider a long pipe of inner radius r_1, outer radius r_2, and thermal conductivity *k*. Fluid is passing through the pipe T_i with a heat transfer coefficient of $\bar{h}_i$ The outer surface of the pipe is subjected to convection to a medium at T_∞ with a heat transfer coefficient of $\bar{h}_0$. Assume that uniform energy $\dot{q}'''$ is generated within the pipe wall. Write down the



conditions.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \cancel{\dot{E}_{st}}^{\,0}$$



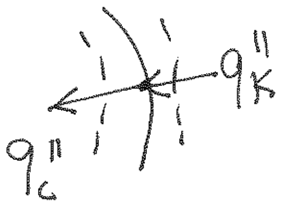
$$-\frac{d}{dr}(Aq_r'') + (2\pi r)(L)\dot{q}''' = 0$$

$$q_r'' = -k\frac{dT}{dr} \qquad A = (2\pi r)(L)$$

So

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}'''}{k} = 0$$

inner surface

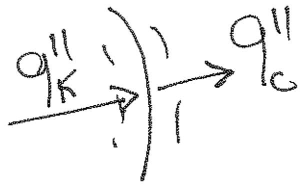


$$\dot{E}_{in} = \dot{E}_{out}$$

$$q''_k = q''_c$$

$$k\left(\frac{dT}{dr}\right)_{r=r_i} = \bar{h}_i\left[T(r_i) - T_\infty\right]$$

external surface



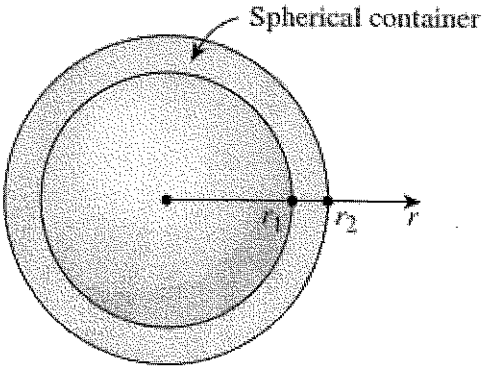
$$\dot{E}_{in} = \dot{E}_{out}$$

$$q''_k = q''_c$$

$$-k\left.\frac{dT}{dr}\right|_{r=r_o} = h\left[T(r_o) - T_\infty\right]$$

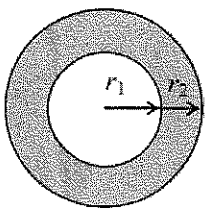
2)

Consider a spherical container of inner radius r_1, outer radius r_2, and thermal conductivity k. Express the boundary condition on the inner surface of the container for steady one-dimensional conduction for the following cases: (a) specified temperature of 50°C, (b) specified heat flux of 45 W/m^2 toward the center, (c) convection to a medium at T_∞ with a heat transfer coefficient of h.



Assume steady state conditions

A spherical container of inner radius r_1, outer radius r_2, and thermal conductivity k is given. The boundary condition on the inner surface of the container for steady one-dimensional conduction is to be expressed for the following cases:



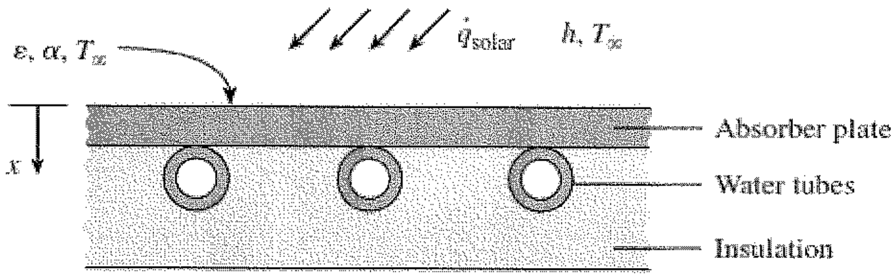
(a) Specified temperature of 50°C: $T(r_1) = 50°C$

(b) Specified heat flux of 45 W/m^2 towards the center: $k\dfrac{dT(r_1)}{dr} = 45\ \text{W/m}^2$

(c) Convection to a medium at T_∞ with a heat transfer coefficient of h: $k\dfrac{dT(r_1)}{dr} = h[T(r_1) - T_\infty]$

3)

A flat-plate solar collector is used to heat water by having water flow through tubes attached at the back of the thin solar absorber plate. The absorber plate has an emissivity and an absorptivity of 0.9. The top surface ($x = 0$) temperature of the absorber is $T_0 = 35°C$, and solar radiation is incident on the absorber at 500 W/m^2 with a surrounding temperature of 0°C. Convection heat transfer coefficient at the absorber surface is 5 W/m^2·K, while the ambient temperature is 25°C. Show that the variation of temperature in the absorber plate can be expressed as $T(x) = -(\dot{q}_0/k)x + T_0$, and determine net heat flux $\dot{q}_0$ absorbed by the solar collector.

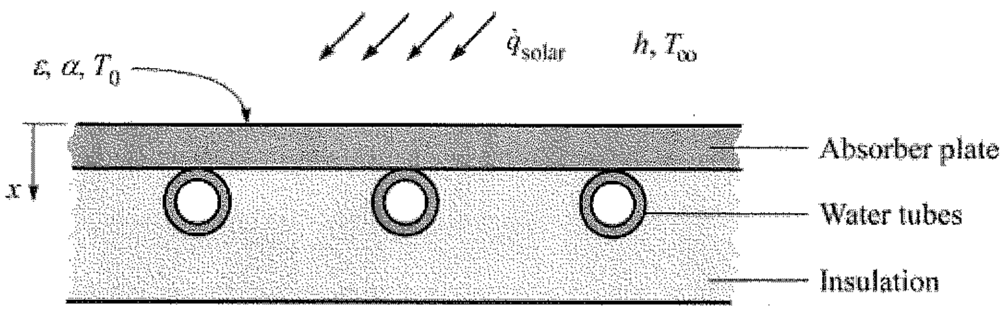


Assume steady state conditions.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** The top surface at $x = 0$ is subjected to convection, radiation, and incident solar radiation.

Properties The absorber surface has an absorptivity of 0.9 and an emissivity of 0.9.

Analysis Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the top surface, the mathematical formulation can be expressed as



Aq''_x

$Aq''_x + \frac{d}{dx}(Aq''_x)dx$

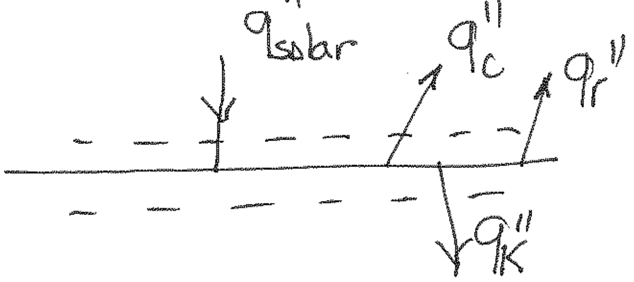
$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g} = \cancel{\dot{E}_{st}}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$-\frac{d}{dx}(Aq''_x) = 0$$

$$q''_x = -k\frac{dT}{dx}$$

$$\frac{d^2T}{dx^2}=0$$



$$\dot{E}_{in}=\dot{E}_{out}$$

$$\alpha q''_{solar}= q''_c+q''_r+q''_k$$

or

$$q''_k= \alpha q''_{solar} - q''_c - q''_r$$

$$-k\left(\frac{dT}{dx}\right)_{x=0} = \underbrace{\alpha q''_{solar} - \bar{h}(T_0-T_\infty) - \epsilon\sigma\left[T_0^4-T_{sur}^4\right]}_{q''_0}$$

$$-k\left(\frac{dT}{dx}\right)_{x=0} = q''_0$$

$$q''_0 = 224 \ W/m^2$$

$x=L$ $\quad T=T_1$ But the problem statement tells us that thickness is negligible and therefore $T_1=T_0$ so $x=0$ [since $L\approx 0$] $T=T_0$

since $T= C_1x+C_2$

$\therefore$ $C_1=-\dfrac{\dot{q}'''_0}{K}$ and $C_2= T_0$

$$T(x)= -\frac{q''_0}{K}x+ T_0$$

4)

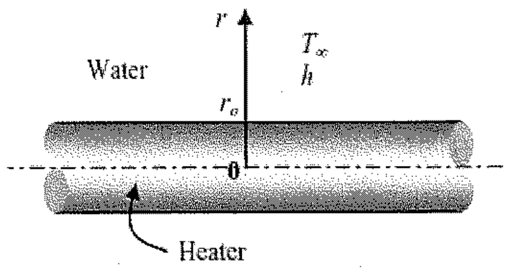
A long homogeneous resistance wire of radius r_o = 0.6 cm and thermal conductivity k = 15.2 W/m·K is being used to boil water at atmospheric pressure by the passage of electric current. Heat is generated in the wire uniformly as a result of resistance heating at a rate of 16.4 W/cm³. The heat generated is transferred to water at 100°C by convection with an average heat transfer coefficient of h = 3200 W/m²·K. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the wire, (*b*) obtain a relation for the variation of temperature in the wire by solving the differential equation, and (*c*) determine the temperature at the centerline of the wire. *Answer:* (c) 125°C

Assume steady state conditions.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 15.2 W/m·K.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{gen}}{k}=0$$

q'''

and $$-k\frac{dT(r_o)}{dr}=h[T(r_o)-T_\infty]$$ (convection at the outer surface)

$$\frac{dT(0)}{dr}=0$$ (thermal symmetry about the centerline)