CHAPTER 8

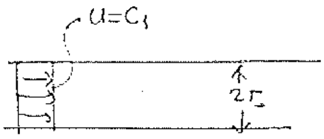
EXAMPLES

1. For flow of liquid metal through a circular tube, the velocity and temperature profiles at a particular axial location may be approximated as being:

$$u(r) = c_1$$
$$T(r) - T_s = c_2\left[1 - r^2/r_o^2\right]$$

What is the value of Nu_D at this location?

Solution



$$h = \frac{q''}{T_s - T_m} \quad ; \quad u_m = \frac{2}{r_0^2}\int_0^{r_0} u r\,dr = C_1$$

$$T_m = \frac{2}{u_m r_0^2}\int_0^{r_0} u T r\,dr$$

$$= \frac{2}{u_m^2 r_0^2}\int_0^{r_0} (C_1)\left\{T_s + C_2(1 - r^2/r_0^2)\right\} r\,dr$$

$$= T_s + \frac{C_2}{2}$$

$$q'' = k\left(\frac{\partial T}{\partial r}\right)_{r=r_0} = -2C_2\frac{k}{r_0}$$

$$\therefore \quad h = \frac{-2C_2(k/r_0)}{-C_2/2} = \frac{4k}{r_0}$$

$$Nu_D = \frac{hD}{k} = \frac{(4k/r_0)(2r_0)}{k} = 8$$

P.2 Corrected solution

A system for heating water from an inlet temperature of $T_{m,i} = 20°C$ to an outlet temperature of $T_{m,o} = 60°C$ involves passing the water through a thick-walled tube having inner and outer diameters of 20 and 40 mm. The outer surface of the tube is well insulated, and electrical heating within the wall provides for a uniform generation rate of $\dot{q} = 10^6$ W/m³.

1. For a water mass flow rate of $\dot{m} = 0.1$ kg/s, how long must the tube be to achieve the desired outlet temperature?
2. If the inner surface temperature of the tube is $T_s = 70°C$ at the outlet, what is the local convection heat transfer coefficient at the outlet?

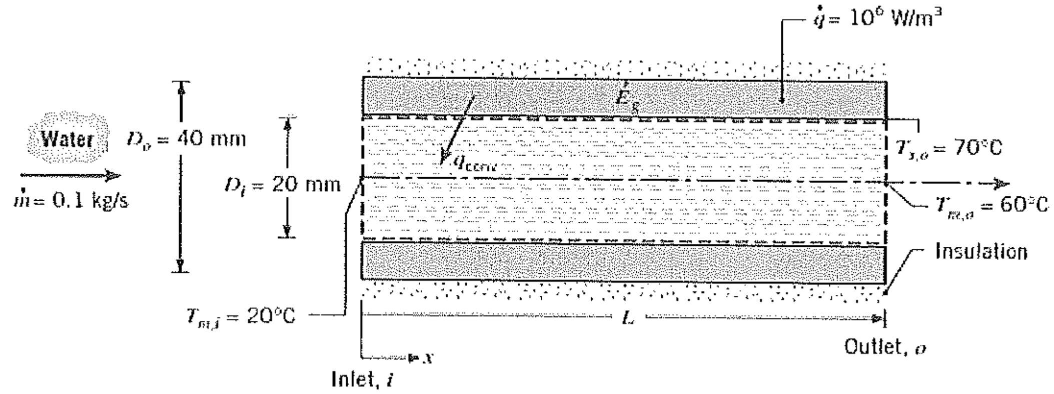
SOLUTION

Known: Internal flow through thick-walled tube having uniform heat generation.

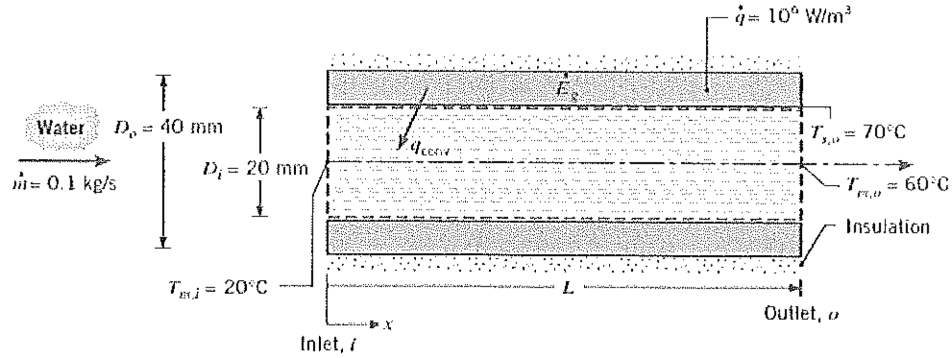
Find:

1. Length of tube needed to achieve the desired outlet temperature.
2. Local convection coefficient at the outlet.

Schematic:



Schematic:



Assumptions:

1. Steady-state conditions.
2. Uniform heat flux.
3. Incompressible liquid and negligible viscous dissipation.
4. Constant properties.
5. Adiabatic outer tube surface.

Properties: Table A.6, water ($\bar{T}_m = 313$ K): $c_p = 4179$ J/kg·K.

Analysis:

1. Since the outer surface of the tube is adiabatic, the rate at which energy is generated within the tube wall must equal the rate at which it is convected to the water.

$$\dot{E}_g = q_{conv}$$

With

$$\dot{E}_g = \dot{q}\frac{\pi}{4}(D_o^2 - D_i^2)L$$

it follows from Equation 8.34 that

$$\dot{q}\,\frac{\pi}{4}\,(D_o^2 - D_i^2)\,L = \dot{m}c_p(T_{m,o} - T_{m,i})$$

or

$$L = \frac{4\dot{m}c_p}{\pi(D_o^2 - D_i^2)\dot{q}}(T_{m,o} - T_{m,i})$$

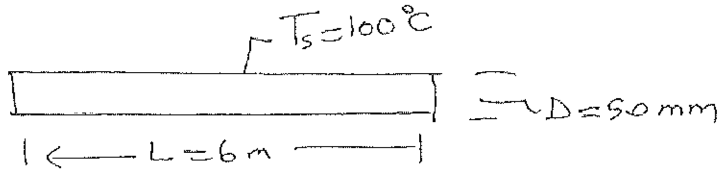
$$L = \frac{4 \times 0.1\text{ kg/s} \times 4179\text{ J/kg}\cdot\text{K}}{\pi(0.04^2 - 0.02^2)\text{ m}^2 \times 10^6\text{ W/m}^3}(60 - 20)^\circ\text{C} = 17.7\text{ m} \qquad \triangleleft$$

2. From Newton's law of cooling, Equation 8.27, the local convection coefficient at the tube exit is

$$h_o = \frac{q_s''}{T_{s,o} - T_{m,o}}$$

3. Steam condensing on the outer surface of a thin walled circular tube of diameter D = 50 mm and length L = 6m maintains a uniform outer surface temperature of 100°C. Water flows through the tube at a rate of $\dot{m} = 0.25\,kg/s$ and its inlet and outlet temperatures are T_{mi} = 15°C and T_{mo} = 57°C. What is the convection coefficient associated with the water flow?

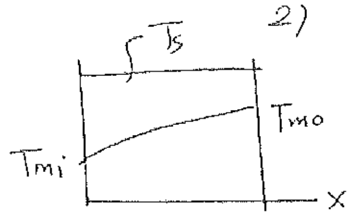
solution



$T_{mi} = 15°C$
$T_{mo} = 57°C$

1) $q_c = \dot{m} c_p (T_{mo} - T_{mi})$

2) $q_c = \bar{h} A_s \Delta T_{LM}$



$$\Delta T_{LM} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

$$\therefore \bar{h} = \frac{\dot{m} c_p (T_{mo} - T_{mi})}{\pi D L \; \Delta T_{LM}}$$

$$\Delta T_{LM} = \frac{(100-57) - (100-15)}{\ln\left(\frac{100-57}{100-15}\right)} = 61.6°C$$

$\bar{h} = 755\ W/m^2 °C$

Let us check this with empirical equations.

1) Dittus Boelter equation

$\bar{T}_m = \frac{T_{mi} + T_{mo}}{2} = 36°C = 309\,K$

$c_p = 4.178\ kJ/kg\,K \qquad \mu = 695 \times 10^{-6}\ Ns/m^2$

$Pr = 4.62 \qquad k = 0.628\ W/mK$

$\bar{h} = \frac{k}{D}\left[0.023\, Re_D^{0.8}\, Pr^{0.4}\right]$

$Re_D = \frac{4\dot{m}}{\pi D \mu} = 9159$ turbulent flow

$\bar{h} = 791\ W/m^2 K$

$$f = \frac{1}{\left[0.79 \ln(Re_D) - 1.64\right]^2}$$

$$= 0.03226$$

$$\overline{Nu}_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7\left[\sqrt{\frac{f}{8}}\left(Pr^{2/3} - 1\right)\right]} = 62.54$$

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = 779 \ W/m^2K$$

4. Ethylene glycol at 60°C with a velocity of u_m = 4 cm/s enters the 6 m long heated section of a thin walled 2.5 cm ID tube, after passing through an isothermal calming section. In the heated part, the wall of the tube is maintained at a uniform temperature of T_s = 100°C by condensing steam on outer surface of the tube. Calculate the exit temperature of ethylene glycol.

Solution

Fluid exit temperature is not known, so the film temperature

$$\bar{T}_f = \frac{1}{2}(T_{mi} + T_{me})$$

can not be calculated. Evaluate physical properties at

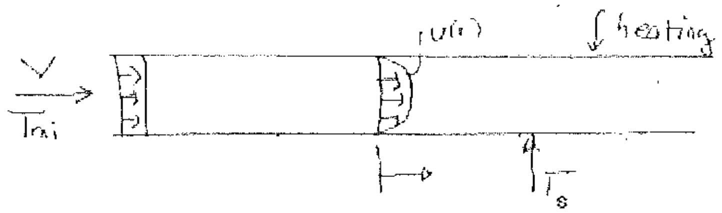
$$T_{mi} = 60°C.$$

$C_p = 2562$ J/kg °C $\qquad \rho = 1088$ kg/m^{-3}

$\nu = 4.75 \times 10^{-6}$ m^2/s $\qquad k = 0.26$ W/m °C

$Pr = 51$

$Re_D = \frac{u_m D}{\nu} = 210$ $\qquad$ flow is laminar



Hydrodynamically fully developed, thermally developing, because of isothermal calming section.

$x_{fd,h} \ll x_{fd,t}$ since Pr is high

i.e. $x_{fd,h} = 0.05 Re_D = (0.05)(210)\left(\frac{2.5}{100}\right) = 0.2625$ m

$x_{fd,t} = 0.05 Re Pr = (0.05)(210)(51) = 13.38$ m

$x_{fd,h} \ll 6$ m ← hydrodynamically fully developed

$x_{fd,h,t} > 6$ m thermally developing flow

$$\frac{x/D}{Re_D Pr} = \frac{600/25}{(210)(\ .7\ )} = 0.0224$$

Mean Nusselt number $\overline{Nu}_D$

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = 5.5$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = 57.2 \quad W/m^2 C$$

$$\bar{h}(\pi D L)\,\Delta T_{LM} = \dot{m}\,c_p\,(T_{mo} - T_{mi})$$

$$\Delta T_{LM} = \frac{\Delta T_i - \Delta T_o}{\ln(\Delta T_i / \Delta T_o)} = \frac{(T_{mo} - T_{mi})}{\ln\left[\frac{T_s - T_{mi}}{T_s - T_{mo}}\right]}$$

$$\dot{m} = \rho V \frac{\pi D^2}{4}$$

after algebra

$$\frac{T_s - T_{mi}}{T_s - T_{mo}} = \exp\left[\frac{4L\bar{h}}{c_p(\rho V_m D)}\right]$$

$$T_{mo} = 76.5 \,^\circ C$$

Now improve computation.
Evaluate properties at $T_f = \frac{76.5 + 60}{2} = 68\,C$
Repeat calculation
to get

$$T_{mo} = 75.4 \,^\circ C$$

close enough

5. In the final stages of a production, pharmaceutical is sterilized by heating it from ~~25°C to 75°C as it moves at 0.2 m/s through a straight thin walled stainless steel~~ tube of 12.7 mm diameter. A uniform heat flux is maintained by electric resistance heater wrapped around the outer surface of tube. If the tube is 10 m long what is the required heat flux? If the fluid enters the tube with a fully developed velocity profile and a uniform temperature profile, what is the surface temperature at the tube exit and a distance of 0.5 m from the entrance? Fluid properties are approximated as: $\rho = 1000\,\mathrm{kg/m^3}, c_p = 4000\,\mathrm{kJ/kg.K}$, $\mu = 2\times10^{-3}\,\mathrm{kg/m.s}$, $k = 0.8\,\mathrm{W/m.K}$, $Pr = 10$. (Use figure given below)

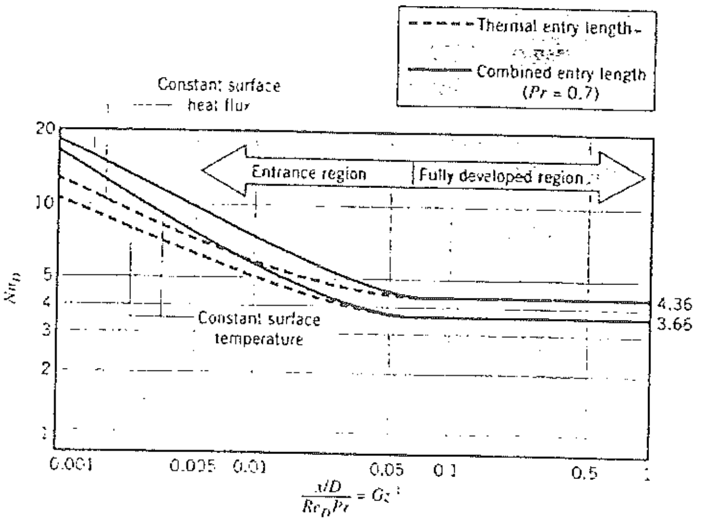
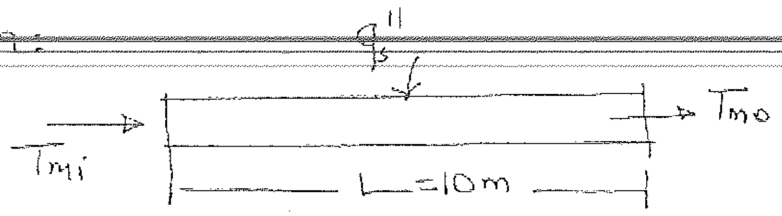


FIGURE 8.9 Local Nusselt number obtained from entry length solutions for laminar flow in a circular tube [2]. Adapted with permission.

Solution:



$T_{mi} = 25°C$ $T_{mo} = 75°C$

$V = u_m = 0.2\ m/s$

$$\dot{m} = \rho VA = \rho V \frac{\pi D^2}{4} = 0.0253\ kg/s$$

$$q = \dot{m} c_p (T_{mo} - T_{mi}) = 5060\ W$$

Heat flux q_s''

$$q_s'' = \frac{q_s}{A_s} = \frac{q_s}{\pi D L} = 12\ 682\ W/m^2$$

$$Re_D = \frac{\rho V D}{\mu} = 1270 \qquad \text{laminar flow}$$

$$x_{fd,t} = 0.05 Re_D Pr = 0.05(1270)(10)(0.0127)$$

$$= 8.06 m > 10 m$$

We have hydrodynamically and thermally fully developed flow at pipe exit.

At $x = 10$ m

$$\bar{h} = \frac{k}{D}\,\overline{Nu}_D$$

$$\overline{Nu}_D = 4.36$$

$$\bar{h} = 4.36\left(\frac{0.8}{0.0127}\right) = 274.6 \quad W/m^2 K$$

$$q_s'' = \bar{h}\,(T_{so} - T_{mo})$$

$$T_{so} = T_{mo} + \frac{q_s''}{\bar{h}} = 121\ °C$$

b) At $x = 0.5\ m < X_{fd,t}$ flow is thermally developing. Thermal entrance problem

From the figure above

$$\left(\frac{x}{D}\right) / Re_D Pr = Gz^{-1} = \frac{(0.5/0.0127)}{(1270)(10)} = 0.0031$$

so $Nu_D \cong 8$

for a thermal entry region with uniform heat flux.

$$Nu_D = \frac{hD}{k} \cong 8$$

$$h = 503.9\ W/m^2 °C$$

$$T_{mo} = T_{mi} + (T_{mo} - T_{mi})\,(x/L) = 27.5\ °C$$

$$T_s = 27.5 + (12682/503.9) = 52.7\ °C$$

6. Air at 1 atm and 27 °C enters a 5.0 mm diameter smooth tube with a velocity of 3 m/s. The length of the tube is 10 cm. A constant heat flux is imposed on the tube wall. Calculate the heat transfer if the exit bulk temperature is 77 °C. Also calculate the exit wall temperature and the value of h at the exit.

Evaluate fluid properties

$$\overline{T_m} = \frac{T_{mi} + T_{mo}}{2} = \frac{27+77}{2} = 52°C = 325K$$

$$\nu = 18.22 \times 10^{-6}\ m^2/s$$

$$Pr = 0.703$$

$$k = 0.02814\ W/mK$$

$$Re_D = \frac{VD}{\nu} = \frac{(3)(5/1000)}{18.22 \times 10^{-6}} = 829$$

Compute entrance lengths

$$X_{fd,h} = 0.05\, Re_D D = 0.05(829)\left(\frac{5}{1000}\right)$$
$$= 0.207 m$$

so $X_{fd,h} > L$ $\quad L = 0.1m$

$$X_{fd,t} = 0.05\, Re_D Pr D = 0.05(829)(0.703)(5/1000)$$
$$= 0.145 m$$

$X_{fd,t} >$

Flow is hydrodynamically and thermally developing. Use figure on next page. Combined entrance length.

Compute Graetz number

$$Gz^{-1} = \frac{X/D}{Re_D Pr} = \frac{(0.1/0.005)}{(829)(0.703)} = 0.0346$$

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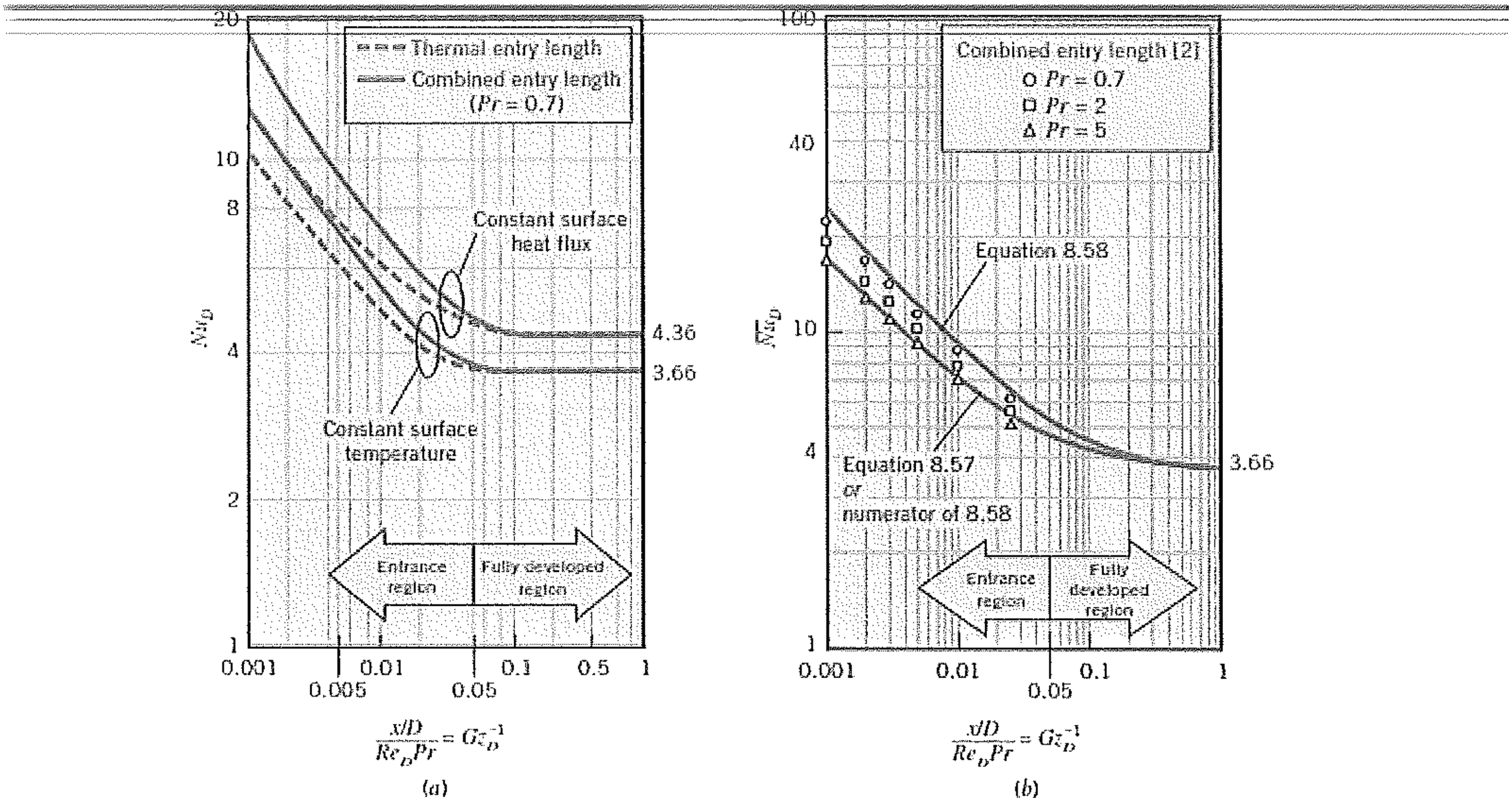


FIGURE 8.10 Results obtained from entry length solutions for laminar flow in a circular tube with constant surface temperature: (*a*) Local Nusselt numbers. (*b*) Average Nusselt numbers.

So $\overline{Nu_D} = \frac{\bar{h} D}{K} \simeq 4.7$

$$\overline{Nu_D} = \frac{q_w'' D}{(T_w - T_m) K}$$

total heat transfer is

$$q = \dot{m} C_p (T_{mo} - T_{mi})$$
$$= (\rho V \tfrac{\pi}{4} D^2) C_p (T_{mo} - T_{mi})$$
$$= 3.49 \text{ W}$$

and heat flux q_w''

$$q_w'' = \frac{q}{A_s} \Rightarrow q = (\pi D L) q_w''$$

$$3.49 = (\pi)\left(\frac{5}{1000}\right)(0.1)\, q_w''$$

$q_w'' = 2222 \text{ W/m}^2$

$$\overline{Nu}_D = \frac{q_w'' D}{(T_w - T_m) K} \Rightarrow \left(q_w'' D / K \overline{Nu}_D \right) = T_w - T_m$$

So $T_w - T_m = \dfrac{(2222)(0.005)}{(4.17)(0.02814)} = 84°C$

$T_w \big|_{x=L} = 84 + 77 = 161°C$

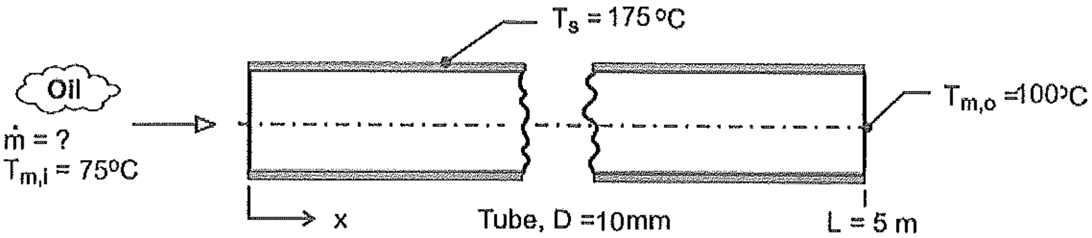
Local heat transfer coefficient

$$h = \frac{q_w''}{[T_w - T_m]_{x=L}} = \frac{2222}{84} = 26.45 \text{ W/m}^2\,°C$$

7.) An oil preheater consists of a single tube of 10 mm diameter and 5-m length with its surface is maintained at 175 ^{0}C by swirling combustion gases. The engine oil(new) enters at 75 ^{0}C .What flow rate must be supplied to maintain an outlet temperature of 100 ^{0}C . What is the corresponding heat transfer rate? Assume that flow is laminar and experimental observations indicate that the flow is both hydrodynamically and thermally developing together.

Solution

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow, (2) Tube wall is isothermal, (3) Incompressible liquid with negligible viscous dissipation, (4) Constant properties.

PROPERTIES: *Table A-5*, Engine oil, new ($T_m = (T_{m,i} + T_{m,o})/2 = 361$ K): $\rho = 847.5$ kg/m^3, c_p = 2163 J/kg·K, $\nu = 2.931 \times 10^{-5}$ m^2/s, k = 0.1379 W/m·K, Pr = 390.2, $\mu = 0.0245$.

Not knowing the mass flow rate , we can not calculate the Reynolds number.

We will flow the following steps:

1) Reynolds number

$$Re_D = \frac{4\dot{m}}{\pi \mu D}$$

2) Since flow is in combined entrance region we can use the following equation

$$\overline{Nu_D} = \frac{\dfrac{3.66}{\tanh[2.264\, Gz_D^{-1/3} + 1.7\, Gz_D^{-2/3}]} + 0.0499\, Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\, Pr^{1/6}\, Gz_D^{-1/6})}$$

$$\begin{bmatrix} T_s = \text{constant} \\ \text{combined entry length} \\ Pr \gtrsim 0.1 \end{bmatrix}$$

3) $\bar{h} = \frac{k}{D}\overline{Nu_D}$

4) We write following equation log mean temperature difference

$$\Delta T_{LMTD} = \frac{\Delta T_i - \Delta T_0}{\ln\left(\frac{\Delta T}{\Delta T_0}\right)}$$

$$\Delta T_i = T_w - T_{mi}$$

$$\Delta T_0 = T_w - T_{m0}$$

5) Energy balance

$$\dot{m}\, c_p\left(T_{m0} - T_{mi}\right) = \bar{h}\left(\pi DL\right)\Delta T_{LMTD}$$

We now have an equation in mass flow rate $\dot{m}$ .Now see the solution by MAPLE 2017

> **restart;**
>
> **k := 0.1379; # Thermal conductivity**

$$k := 0.1379$$

> **L := 5; # Tube length**

$$L := 5$$

> **d := $\frac{10}{1000.0}$; # D=d tube diameter**

$$d := 0.01000000000$$

> **μ := 0.0252;**

$$\mu := 0.0252$$

> **cp := 2163;**

$$cp := 2163$$

> **Pr := 390;**

$$Pr := 390$$

> **Gz := $\left(\frac{d}{L}\right)\cdot\left(\frac{4\cdot m}{\pi\cdot d\cdot\mu}\right)\cdot Pr$;**

$$Gz := 3940.979542\ m$$

> **A := $\frac{3.66}{\tanh\left(2.264\cdot Gz^{-\frac{1}{3}} + 1.7\cdot Gz^{-\frac{2}{3}}\right)}$;**

$$A := \frac{3.66}{\tanh\left(\frac{0.1433315164}{m^{1/3}} + \frac{0.006813644455}{m^{2/3}}\right)}$$

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> $B := 0.04999 \cdot Gz \cdot \tanh\left(\frac{1}{Gz}\right);$

$$B := 197.0095673\ m \tanh\left(\frac{0.0002537440221}{m}\right)$$

> $C := \tanh\left(2.432 \cdot Pr^{\frac{1}{6}} \cdot Gz^{-\frac{1}{6}}\right);$

$$C := \tanh\left(\frac{0.6119221971\ 390^{1/6}}{m^{1/6}}\right)$$

> $h := \left(\frac{k}{d}\right) \cdot \left(\frac{(A + B)}{C}\right);$

$$h := \frac{1}{\tanh\left(\frac{0.6119221971\ 390^{1/6}}{m^{1/6}}\right)} \left(13.79000000 \left(3.66 \Big/ \left(\tanh\left(\frac{0.1433315164}{m^{1/3}} + \frac{0.006813644455}{m^{2/3}}\right)\right) + 197.0095673\ m \tanh\left(\frac{0.0002537440221}{m}\right)\right)\right)$$

> $\Delta T_{LMTD} := \frac{25.}{\ln\left(\frac{100.}{75}\right)};$

$$\Delta T_{LMTD} := 86.90148750$$

> $T_{mo} := 100;$

$$T_{mo} := 100$$

> $T_{mi} := 75;$

$$T_{mi} := 75$$

> # m is mass flow rate

>

>

> $eq := m \cdot cp \cdot \left(T_{mo} - T_{mi}\right) - h \cdot \pi \cdot (d) \cdot L \cdot \Delta T_{LMTD};$

$$eq := -\frac{1}{\tanh\left(\frac{0.6119221971\ 390^{1/6}}{m^{1/6}}\right)} \left(188.2397570 \left(3.66 \Big/ \left(\tanh\left(\frac{0.1433315164}{m^{1/3}} + \frac{0.006813644455}{m^{2/3}}\right)\right) + 197.0095673\ m \tanh\left(\frac{0.0002537440221}{m}\right)\right)\right) + 54075\ m$$

> $eq1 := \mathrm{evalf}(eq);$

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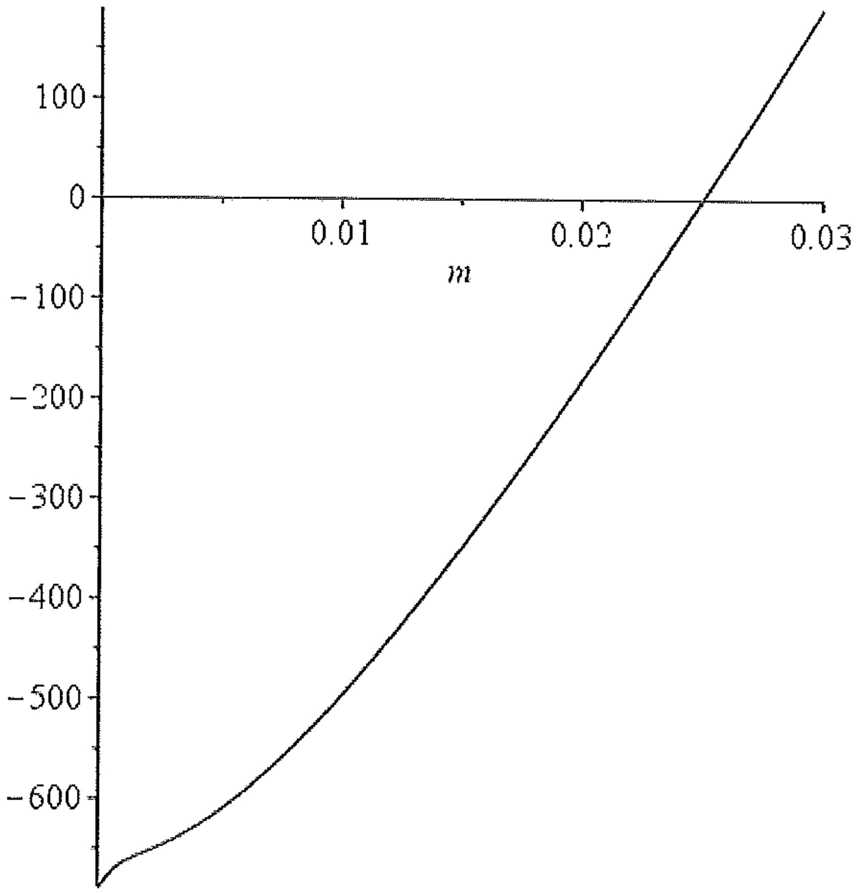
eq1 :=

$$-\frac{1}{\tanh\left(\frac{1.654018293}{m^{1/6}}\right)}\left(188.2397570\left(3.66\left(\tanh\left(\frac{0.1433315164}{m^{1/3}}+\frac{0.006813644455}{m^{2/3}}\right)\right)+197.0095673\, m \tanh\left(\frac{0.0002537440221}{m}\right)\right)\right)+54075.\, m$$

> plot(eq, m = 0 ..0.03);



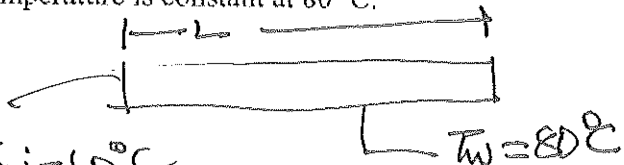
> fsolve(eq1, m);

0.02501504890

>

>

8. Water at 60 °C enters a tube of 1 in (2.54 cm) diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3 m long and the wall temperature is constant at 80 °C.



$D = 1'' = 2.54$ cm

$L = 3$ m

$T_{mi} = 60°C$

$V = 2$ cm/s

evaluate properties at 60°C

$c_p = 4.18$ kJ/kg °C

$Pr = 3.02$ $\rho = 985$ kg/m^3

$\mu = 4.71 \times 10^{-4}$ kg/m.s $k = 0.651$ W/m °C

$$Re_D = \frac{VD\rho}{\mu} = \frac{(985)(0.02)(0.0254)}{4.71 \times 10^{-4}} = 1062$$

$X_{fd,h} = 0.05\, Re_D D = (0.05)(1062)(0.0254) = 1.34$ m

$X_{fd,h} < L$; flow is hydrodynamically developed.

$X_{fd,t} = 0.05\, Re_D D\, Pr = 0.05(1062)(0.0252)(3.02)$

$= 4.07$

$X_{fd,t} > L$ flow is thermally developing

This is thermal entrance problem.

Use figure B.10 of textbook

$$Gz_D^{-1} = \frac{X/D}{Re_D Pr} = \frac{L/D}{Re_D Pr} = \frac{3/0.0254}{(1062)(3.02)} = 0.0368$$

$Gz = 27.17$

or you can use

$$\overline{Nu}_D = 3.66 + \frac{0.0668\left[\frac{D}{L} Pe\right]}{1 + 0.04\left[\frac{D}{L} Pe\right]^{2/3}}$$

$Pe = Re_D Pr$

Using given equation

$$\overline{Nu_D} \cong 5 \rightarrow \bar{h} = \frac{k}{D}\overline{Nu_D} = \left(\frac{0.651}{0.0254}\right)(5)$$

$$\bar{h} \cong 128 \ W/m^2 {}^\circ C$$

$$\frac{T_s - T_{mo}}{T_s - T_{mi}} = \exp\left[-\frac{\bar{h} P L}{\dot{m} c_p}\right]$$

$$\dot{m} = \rho V A = \rho V \frac{\pi D^2}{4} = (985)\left(\frac{2}{100}\right)\left(\frac{\pi}{4}\right)(0.0254)^2$$

$$= 0.009982 \ kg/s$$

$$P = \pi D = (\pi)(0.0254) = 0.07979$$

$$T_{mo} = 70.4^\circ C \approx 70^\circ C$$

Now evaluate mean temperature $\overline{T_m}$

$$\overline{T_m} = \frac{1}{2}(60 + 70) = 65^\circ C$$

Evaluate fluid properties $65^\circ C$

$\rho = 979 \ kg/m^3$ $\quad c_p = 4.188 \ kJ/kg\,K$

$\mu = 420 \times 10^{-6} \ N.s/m^2$ $\quad k = 0.660 \ W/mK$

$Pr = 2.66$

$$Re_D = \frac{\rho V D}{\mu} = \frac{(979)(0.02)(0.0254)}{420 \times 10^{-6}} = 1184$$

$$X_{fd,h} = 0.05 \, Re_D \, D = 1.503 m$$

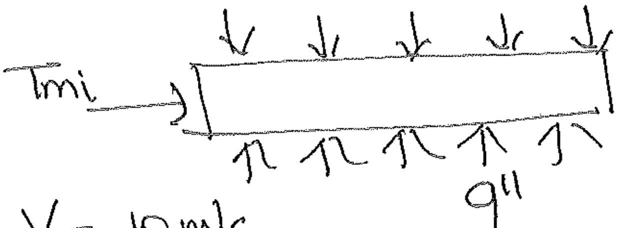
$$X_{fd,h} < L$$

$$X_{fd,t} = 0.05 \, Re_D \, D \, Pr = 4 m$$

$$X_{fd,t} > L$$

Thermal entrance problem

9. Air at 2 atm and 200 °C is heated as it flows through a tube with a diameter of 1 in (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant heat flux condition is maintained at the wall and the wall temperature is 20 °C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3 m length of the tube?



$D = 2.54$ cm

$L = 3$ m

$T_W = T_m + 20$ °C

$V = 10$ m/s

$T_{mi} = 200$ °C

$P = 2$ atm

$$\rho = \frac{P}{RT} = \frac{(2)(1.0325\times10^{5})}{(287)(473)} = 1.493 \text{ kg/m}^3$$

$Pr = 0.681$

$\mu = 2.57\times10^{-5}$ kg/ms

$K = 0.0386$ W/m °C

$C_p = 1.025$ KJ/kg °C

at $\overline{T_m} = 200$ °C

w

$$Re_D = \frac{\rho V D}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57\times10^{-5}} = 14756 > 2300$$

turbulent flow

we may use Dittus Boelter eq

$$\overline{Nu_D} = \frac{\bar{h} D}{K} = 0.023\, Re_D^{0.8}\, Pr^{0.4} = 42.67$$

$$\bar{h} = \frac{K}{D}\overline{Nu_D} = 64.85 \text{ W/m °C}$$

$$\frac{q}{L} = hP(T_W - T_m) = \bar{h}\pi D(T_W - T_m)$$

$$= (64.85)(\pi)(0.0254)(20) = 103.5 \text{ w/m}$$

Next find increase in temperature

$$q = \dot{m} C_p (T_{mo} - T_{mi})$$

$$\Delta T = T_{mo} - T_{mi} = \frac{\left[\frac{q}{L}\right] L}{\dot{m}\, C_p}$$

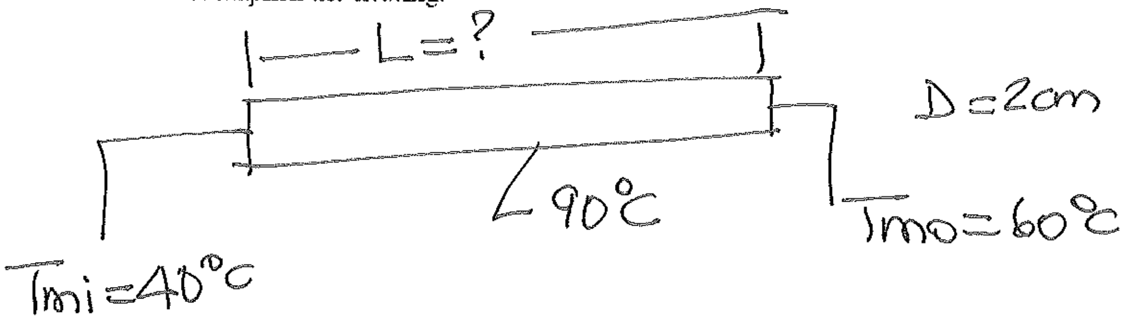
$$\dot{m} = \rho V A = \rho V \frac{\pi D^2}{4} = (1.493)(10)(\pi)\frac{(0.0254)^2}{4}$$

$$= 7.565 \times 10^{-3} \text{ kg/s}$$

$$\text{so } \Delta T = \frac{(103.5)(3)}{(7.565 \times 10^{-3})(1025)} \cong 40.04\ ^\circ C$$

Note: in turbulent flow usually entrance effects can be neglected.

10. A 2.0 cm diameter tube having a relative roughness of 0.001 is maintained at a constant wall temperature of 90 °C. Water enters the tube at 40 °C and leaves at 60 °C. If the entering velocity is 3 m/s, calculate the length of tube necessary to accomplish the heating.



$V = 3\ m/s$

$\bar{T}_m = \frac{1}{2}(T_{mi} + T_{mo}) = \frac{60+40}{2} = 50°C$

Estimate fluid properties at

$T_f = \frac{\bar{T}_m + T_w}{2} = 70°C$ ← this is a better way to estimate fluid properties

$\rho = 978\ kg/m^3$

$\mu = 4 \times 10^{-4}\ kg/ms$

$k = 0.664\ W/m°C$

$Pr = 2.54$

$c_p = 4188\ J/kg°C$

$\mu_s = 2.81 \times 10^{-4}$ at $100°C$

$Re_D = \frac{\rho V D}{\mu} = \frac{(978)(3)(0.02)}{4 \times 10^{-4}} = 146700 > 2300$

turbulent flow

we will use

$$Nu_D = \frac{(f/8)[Re_D - 1000]Pr}{1 + 12.7\sqrt{\frac{f}{8}}(Pr^{2/3} - 1)}$$

$\frac{\epsilon}{D} = 0.001$

using ~~Moody chart. or~~

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right)$$ with fzero

command of Matlab or fsolve
command of Maple 2017
see next page for Maple 2017
solution.

$$f = 0.02146$$

$$\overline{Nu_D} = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7\sqrt{\frac{f}{8}}\left(Pr^{2/3} - 1\right)}$$

$$= 633.6$$

$$\bar{h} = \frac{k}{D}\overline{Nu_D} = \left(\frac{0.664}{0.02}\right)(633.6) = 21036\ W/m^2\,°C$$

$$q = \dot{m}\,c_p\,(T_{mo} - T_{mi})$$

$$q = \bar{h}\pi DL\,(T_w - \bar{T}_m)$$

$$\dot{m}\,c_p\,(T_{mo} - T_{mi}) = \bar{h}\pi DL\,(\bar{T}_w - \bar{T}_m)$$

$$q = \dot{m}c_p\,(T_{mo} - T_{mi})$$

$$= \left(\rho V\frac{\pi D^2}{4}\right)(c_p)(T_{mo} - T_{mi})$$

$$= (978)(3)\left(\frac{\pi}{4}\right)(0.02)^2(4188)(60 - 40)$$

$$= 77020\ W$$

> *restart*;

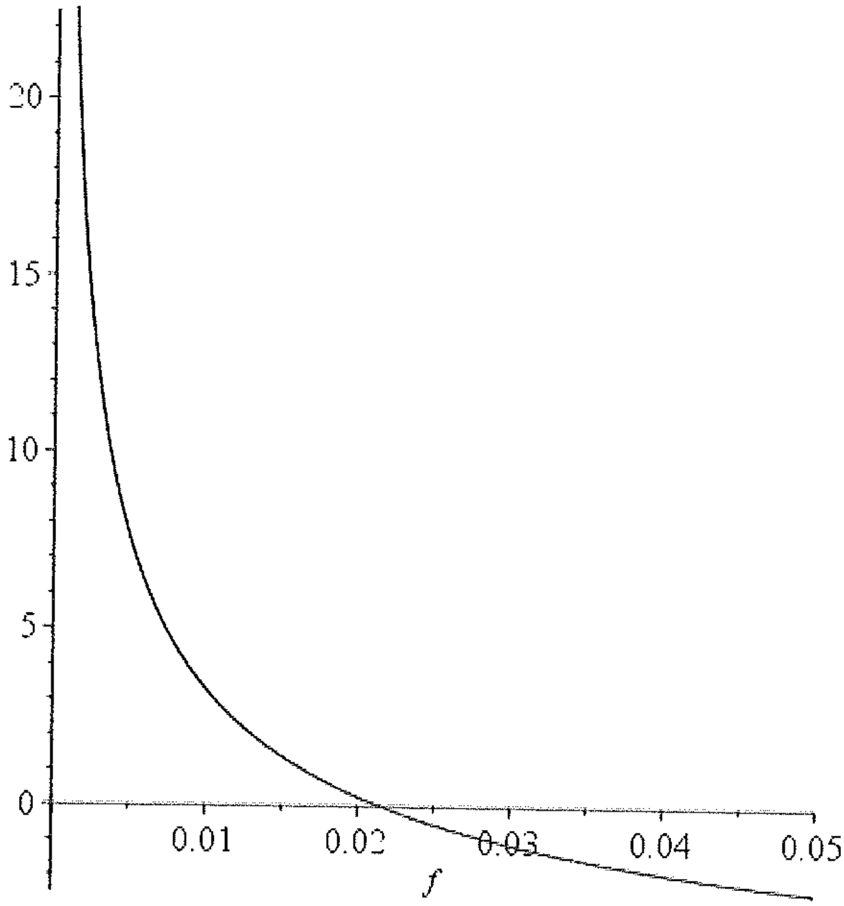
>

> $eq := \frac{1}{\sqrt{f}} + 2.0 \cdot \log 10\left(\frac{\left(\frac{0.001}{1}\right)}{3.7} + \frac{2.5}{146700.0 \cdot \sqrt{f}}\right)$;

$$eq := \frac{1}{\sqrt{f}} + \frac{2.0 \ln\left(0.0002702702703 + \frac{0.00001704158146}{\sqrt{f}}\right)}{\ln(10)}$$

>

> *plot*$(eq, f = 0 ..0.05)$;



> *fsolve*$(eq, f = 0.02 ..0.03)$;

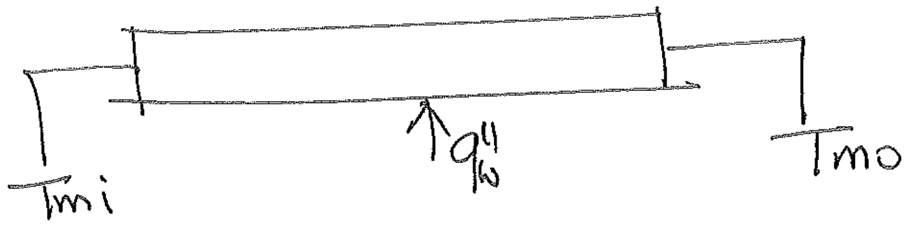
$$0.02146498275$$

$$77020 = (21036)(\pi)(0.02)(L)(90-50)$$

$$L = 1.45\,m$$

25/8

11. Air at 300 K and 1 atm enters a smooth tube having a diameter of 2.0 cm and length of 10 cm. The air velocity is 40 m/s. What constant heat flux must be applied at the tube surface to result in an air temperature rise of 5 °C? What average wall temperature would be necessary for this case?



$T_{mi} = 300K$ $\quad$ $D = 2$ cm

$P = 1$ atm $\quad$ $L = 10$ cm

$V = 40$ m/s $\quad$ $\Delta T = T_{mo} - T_{mi} = 5$ °C

Evaluate fluid properties at $T_{mi} = 300$ K since we don't know exit temperature.

$\nu = 15.69 \times 10^{-6}$ m²/s $\quad$ $C_p = 1006$ J/kg °C

$k = 0.02624$ W/m K $\quad$ $\rho = 1.18$ kg/m³

$Pr = 0.7$

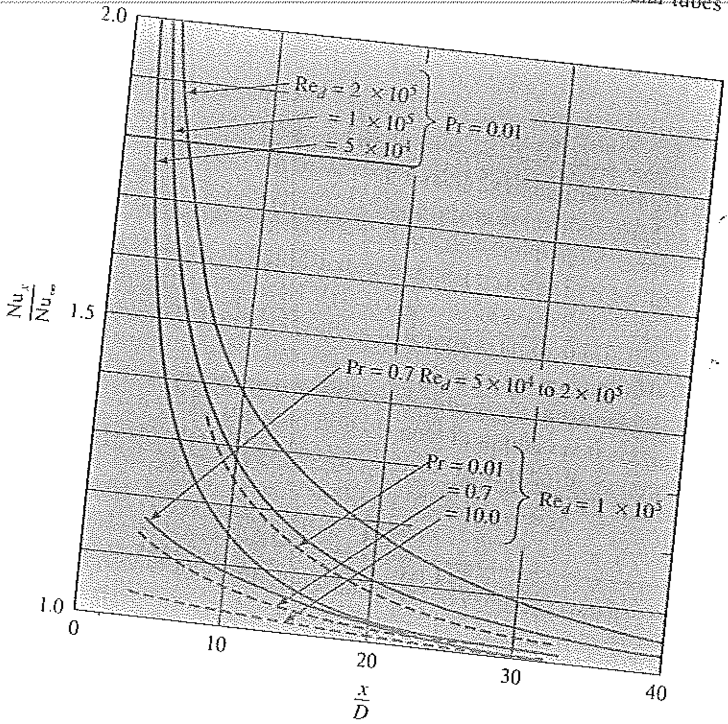
$$Re_D = \frac{VD}{\nu} = \frac{(40)(0.02)}{15.69 \times 10^{-6}} = 50\,988 > 2300$$

turbulent flow

$\frac{L}{D} = \frac{10}{2} = 5$ so we have entrance problem since this number is less than $\frac{L}{D} = 10$.

Use the following figure

Figure 6-6 | Turbulent thermal entry Nusselt numbers for circular tubes with T_w = constant.



$Pr = 0.7$

$\frac{L}{D} = 0.5$

$\frac{Nu_x}{Nu_\infty} \cong 1.15$

so heat transfer is about 15%
higher than thermally developed flow.

Let us use $\overline{Nu_D} = 0.023\, Re_D^{4/5}\, Pr^{1/3}$

$\overline{Nu_D} = 0.023(50988)^{0.8}(0.7)^{0.4} \doteq 116.3$

$\bar{h} = \frac{k}{D}\overline{Nu_D} = 152.6\ W/m^2 K$

now new $\bar{h}$ (i.e. corrected $\bar{h}$)

$\bar{h} = 1.15)(152.6) = 175.5\ W/m^2\,°C$

$\dot{m} = \rho V A = \rho V \frac{\pi D^2}{4} = (1.18)(40)\left(\frac{\pi}{4}\right)(0.02)^2$

$= 0.0148\ kg/s$

27/8

$$q = \dot{m} c_p (T_{mo} - T_{mi})$$

$$= (0.0148)(1006)(5) = 74.4\ W$$

$$A = \pi D L = (\pi)(0.02)(0.1)$$

$$= 0.0628\ m^2$$

$$q'' = \frac{q}{A} = \frac{74.4}{0.0628} = 1184\ W/m^2$$

$$\bar{T}_m = \frac{(300 + 305)}{2} = 302.5\ K$$

$$q'' = \bar{h}(T_w - \bar{T}_m) \longrightarrow \bar{T}_w = \bar{T}_m + \frac{q''}{\bar{h}}$$

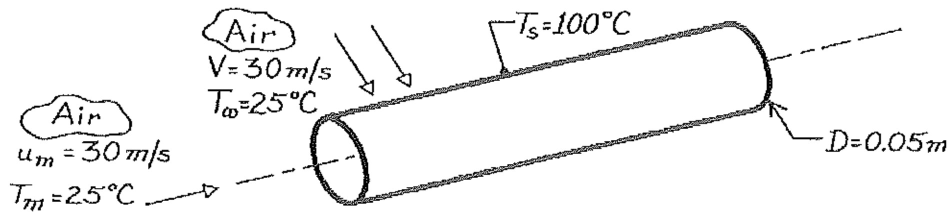
$$\bar{T}_w = 302.5 + \frac{1184}{17.5} = 370\ K$$

12. The surface of a 50 mm diameter, thin-walled tube is maintained at 100°C. In one case air is in cross flow over the tube with a temperature of 25°C and a velocity of 30 m/s. In another case air is fully developed flow through tube with a temperature of 25°C and a mean velocity of 30 m/s. Compare the heat flux from the tube to the air for two cases.

KNOWN: Surface temperature and diameter of a tube. Velocity and temperature of air in cross flow. Velocity and temperature of air in fully developed internal flow.

FIND: Convection heat flux associated with the external and internal flows.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions. (2) Uniform cylinder surface temperature. (3) Fully developed internal flow.

PROPERTIES: *Table A-4*. Air (298K): $\nu = 15.71 \times 10^{-6}\ m^2/s$, $k = 0.0261$ W/m·K, $Pr = 0.71$.

ANALYSIS: For the *external* and *internal* flows.

$$Re_D = \frac{VD}{\nu} = \frac{u_m D}{\nu} = \frac{30\ m/s \times 0.05\ m}{15.71 \times 10^{-6} m^2/s} = 9.55 \times 10^4.$$

From the Zhukauskas relation for the *external* flow, with $C = 0.26$ and $m = 0.6$,

$$\overline{Nu}_D = C\, Re_D^m\, Pr^n\, (Pr/Pr_s)^{1/4} = 0.26\left(9.55 \times 10^4\right)^{0.6} (0.71)^{0.37} (1)^{1/4} = 223.$$

Hence, the convection coefficient and heat flux are

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0261\ W/m \cdot K}{0.05\ m} \times 223 = 116.4\ W/m^2 \cdot K$$

$$q'' = h(T_s - T_\infty) = 116.4 W/m^2 \cdot K(100 - 25)^\circ C = 8.73 \times 10^3\ W/m^2. \qquad <$$

Using the Dittus-Boelter correlation, Eq. 8.60, for the *internal* flow, which is turbulent,

$$\overline{Nu}_D = 0.023\, Re_D^{4/5}\, Pr^{0.4} = 0.023\left(9.55 \times 10^4\right)^{4/5} (0.71)^{0.4} = 193$$

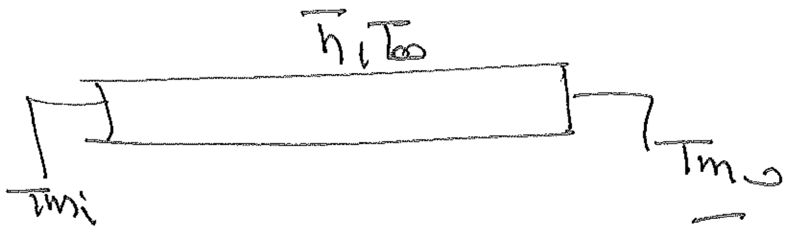
$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0261\ W/m \cdot K}{0.05\ m} \times 193 = 101\ W/m^2 \cdot K$$

and the heat flux is

$$q'' = h(T_s - T_m) = 101\ W/m^2 \cdot K(100 - 25)^\circ C = 7.58 \times 10^3\ W/m^2. \qquad <$$

COMMENTS: Convection effects associated with the two flow conditions are comparable.

13. A thin-walled insulated 0.3 m diameter duct is used to route chilled air at 0.05 kg/s through the attic of a large commerical building. The attic air is at 37 °C and natural circulation provides a convection coefficient of 2 $W/m^2.K$ at the outer surface of the duct. If chilled air enters a 15 m long duct at 7 °C, what are the exit temperature and the rate of heat gain?



evaluate properties at $\bar{T}_m = 280\ K$

$\mu = 170 \times 10^{-7}\ NS/m^2$ $\quad k = 0.024\ W/m\,°C$

$Pr = 0.708$ $\quad c_p = 1007\ J/kg\,°C$

$T_{mi} = 7°C = 280\ K$

$\dot{m} = 0.05\ kg/s$ $\quad T_\infty = 37°C$

$L = 15\ m$ $\quad \bar{h} = 2\ W/m^2\,°C$

$$Re_D = \frac{\rho V D}{\mu} = \frac{4\dot{m}}{\mu \pi D} = \frac{4(0.05)}{(170\times10^{-7})(\pi)(0.3)}$$

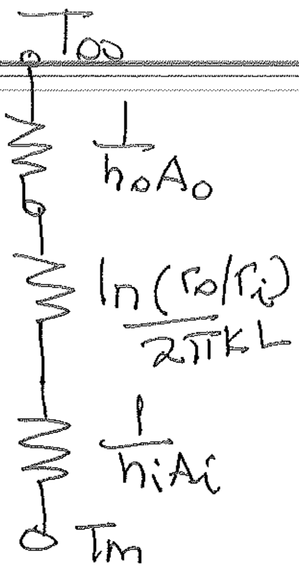
$= 12\ 482 > 2300$ turbulent flow

$$\frac{X_{fdh}}{D} = \frac{15}{0.3} = 50 \quad \text{so} \quad 10 \le \frac{X_{fdh}}{D} \le 60$$

$X_{fdt} = 10D = 10(0.3) = 3\ m < L$

flow is hydrodynamically and thermally fully developed.

$$\frac{T_\infty - T_{mo}}{T_\infty - T_{mi}} = \exp\left[-\frac{PLU}{\dot{m}c_p}\right]$$



we can neglect wall resistance

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

we need h_i. Use Dittus Boelter equation

$$\overline{Nu_D} = 0.023\, Re_D^{0.8}\, Pr^{0.4}$$

$$= 0.023\,(12482)^{0.8}\,(0.708)^{0.4}$$

$$\simeq 38$$

$$h_i = \frac{k}{D}(\overline{Nu_D}) = \left(\frac{0.024}{0.3}\right)(38) \cong 3 \ W/m^2K$$

$$U = \left[\frac{1}{3} + \frac{1}{2}\right]^{-1} \approx 1.2 \ W/m^2K$$

$$\frac{37 - T_{mo}}{37 - 7} = \exp\left[-\frac{(\pi)(0.3)(15)(1.2)}{(0.05)(1007)}\right]$$

$$T_{mo} = 15.7 \ °C$$

b) $q = \dot{m}\, c_p (T_{mo} - T_{mi})$

$= (0.05)(1007)(15.7 - 7) \approx 438 \ W$

Example

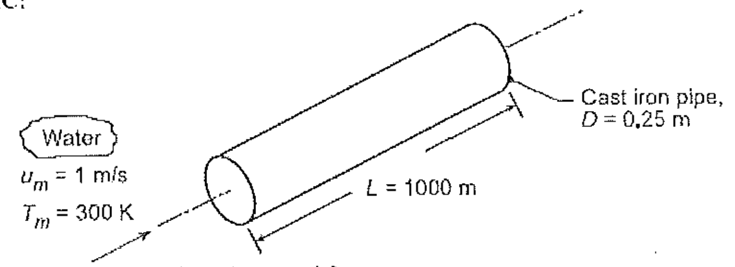
Water at 27 ^{0}C flow with a mean velocity of 1 m/s through a 1000m long pipe of 0.25 m inside diameter:
a) Determine the pressure drop over the pipe length and the corresponding pump power requirement if the pipe surface is smooth.
b) If the pipe is made of cast iron and its surface is clean Determine the pressure drop over the pipe length and the corresponding pump power requirement
c) For the smooth pipe condition generate a plot of pressure drop and pump requirement for the mean velocities in the range from 0.05 to 1.5 m/s.

SOLTION

KNOWN: Temperature and velocity of water flow in a pipe of prescribed dimensions.

FIND: Pressure drop and pump power requirement for (a) a smooth pipe, (b) a cast iron pipe with a clean surface, and (c) smooth pipe for a range of mean velocities 0.05 to 1.5 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, fully developed flow.

PROPERTIES: *Table A6*, Water (300 K): $\rho = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho = 8.576 \times 10^{-7}$ m^2/s.

ANALYSIS: From Eq. 8.22a and 8.22b, the pressure drop and pump power requirement are

$$\Delta p = f\frac{\rho u_m^2}{2D}L \qquad P = \Delta p\dot{V} = \Delta p\left(\pi D^2/4\right)u_m \qquad (1,2)$$

The friction factor, f, may be determined from Figure 8.3 for different relative roughness, e/D, surfaces or from Eq. 8.21 for the smooth condition, $3000 \le Re_D \le 5 \times 10^6$,

$$f = (0.790\ln(Re_D) - 1.64)^{-2} \qquad (3)$$

where the Reynolds number is

$$Re_D = \frac{u_m D}{\nu} = \frac{1\,\text{m/s} \times 0.25\,\text{m}}{8.576 \times 10^{-7}\,\text{m}^2/\text{s}} = 2.915 \times 10^5 \qquad (4)$$

(a) *Smooth surface:* from Eqs. (3), (1) and (2),

$$f = \left(0.790\ln\left(2.915 \times 10^5\right) - 1.64\right)^{-2} = 0.01451$$

$$\Delta p = 0.01451\left(997\,\text{kg/m}^3 \times 1\,\text{m}^2/\text{s}^2 / 2 \times 0.25\,\text{m}\right)1000\,\text{m} = 2.89 \times 10^4\,\text{kg/s}^2\cdot\text{m} = 0.289\,\text{bar} \quad <$$

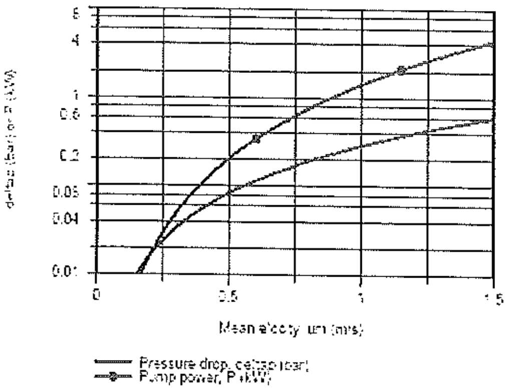
$$P = 2.89 \times 10^4\,\text{N/m}^2\left(\pi \times 0.25^2\,\text{m}^2/4\right)1\,\text{m/s} = 1418\,\text{N}\cdot\text{m/s} = 1.42\,\text{kW} \quad <$$

(b) *Cast iron clean surface:* with e = 260 μm, the relative roughness is e/D = 260 × 10^{-6} m/0.25 m = 1.04 × 10^{-3}. From Figure 8.3 with $Re_D = 2.92 \times 10^5$, find f = 0.021. Hence,

$$\Delta p = 0.402\,\text{bar} \qquad P = 1.97\,\text{kW} \quad <$$

(c) *Smooth surface:* Using IHT with the expressions of part (a), the pressure drop and pump power requirement as a function of mean velocity, u_m, for the range $0.05 \le u_m \le 1.5$ m/s are computed and plotted below.

Continued...



The pressure drop is a strong function of the mean velocity. So is the pump power since it is proportional to both Δp and the mean velocity.

COMMENTS: (1) Note that $L/D = 4000 >> (x_{fd,h}/D) \approx 10$ for turbulent flow and the assumption of fully developed conditions is justified.

(2) Surface fouling results in increased surface roughness and increases operating costs through increasing pump power requirements.

(3) The *IHT Workspace* used to generate the graphical results follows.

```
// Pressure drop:
deltap = f * rho * um^2 * L / ( 2 * D )          // Eq (1); Eq 8.22a
deltap_bar = deltap / 1.00e5                     // Cnersion, Pa to bar units
Power = deltap * ( pi * D^2 / 4 ) * um           // Eq (2); Eq 8.22b
Power_kW = Power / 1000                          // Ueful for scaling graphical result

// Reynolds number and friction factor:
ReD = um * D / nu                                // Eq (3)
f = (0.790 * ln (ReD) - 1.64 ) ^ (-2)            // Eq (4); Eq 8.21, smooth surface condition

// Properties Tool - Water:
// Water property functions T dependence, From Table A.6
// Uts:T(K), p(bars);
x = 0                              // Qality   (0=sat liquid or 1=sat apor)
rho = rho_Tx("Water",Tm,x)         // Density, kg/m^3
nu = nu_Tx("Water",Tm,x)           // Kinematic iscosity , m^2/s

// Assigned variables:
um = 1                             // Mean elocity , m/s
Tm = 300                           // Mean temperature, K
D = 0.25                           // Tube diameter, m
L = 1000                           // Tube length, m
```

Example- Heating of Air in Laminar Tube Flow for Constant Heat Flux:
Air at 1 atm and 27 °C enters a 5.0 mm diameter smooth tube with a velocity of 3 m/s. The length of the tube is 10 cm. A constant heat flux is imposed on the tube wall. Calculate the heat transfer if the exit bulk temperature is 77 °C. Also calculate the exit wall temperature and the value of h at the exit.

We first evaluate the flow regime and do so by taking properties at the average bulk temperature

$$\bar{T}_m = \frac{27+77}{2} = 52°C = 325K$$

$\nu = 18.22 \times 10^{-6}\ m^2/s$

$Pr = 0.703$

$k = 0.02814\ W/m°C$

$$Re_D = \frac{VD}{\nu} = \frac{(3)(0.005)}{(18.22 \times 10^{-6})} = 823$$

so that the flow is laminar. The tube length is rather short so we expect a thermal entrance effect and shall consult Figure 8.9. The inverse Graetz number is computed as

$$Gz^{-1} = \frac{1}{Re_D Pr}\frac{x}{D} = \frac{0.1}{(823)(0.703)(0.005)} = 0.0346$$

Therefore, for q''_s = constant, we obtain the Nusselt number at exit from Figure 8.9 as

$$Nu_D = \frac{hD}{k} = 4.7 = \frac{q''_s D}{(T_s - T_m)k}$$

The total heat transfer is obtained in terms of the overall energy balance:

$$\dot{Q} = \dot{m} c_p (T_{mo} - T_{mi})$$

At entrance $\rho = 1.1774\ kg/m^3$, so the mass flow is

$$\dot{m} = (1.1774)\pi(0.0025)^2(3.0) = 6.94 \times 10^{-5}\ kg/s$$

and

$$\dot{Q} = (6.94\times10^{-5})(1006)(77-27) = 3.49\ W$$

Thus we may find the heat transfer without actually determining wall temperatures or values of $\bar{h}$. However, to determine T_s we must compute $\dot{q}_s''$

$$\dot{Q} = (q_s'')(\pi D L) = 3.49\ W$$

and

$$q_s'' = 2222\ W/m^2$$

$$Nu_D = \frac{q_s'' D}{(T_s - T_m)k} \Rightarrow (T_s - T_m)_{x=L} = \frac{(2222)(0.005)}{(4.7)(0.02814)} = 84\,°C$$

The wall temperature at exit is thus

$$T_s\big|_{x=L} = 84 + 77 = 161\,°C$$

and heat transfer coefficient is

$$\bar{h}\big|_{x=L} = \frac{q_s''}{(T_s - T_m)_{x=L}} = \frac{2222}{84} = 26.45\ W/m^2\,°C$$