# Cankaya University
# Faculty of Engineering
# Mechanical Engineering Department
# ME 313 Heat Transfer

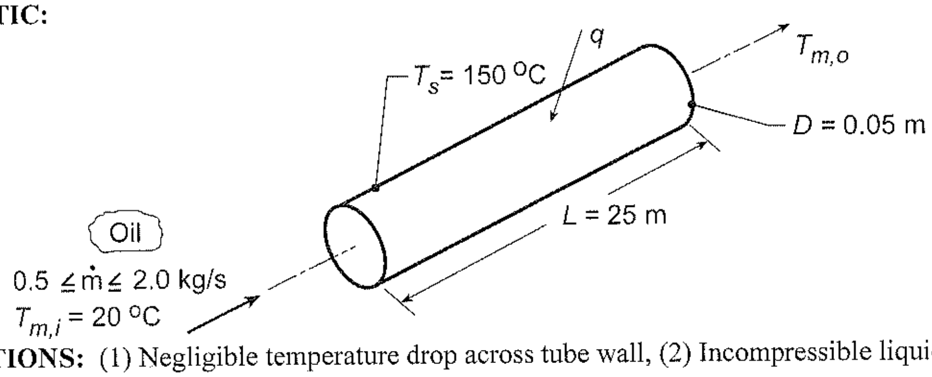
## Chapter 8
## Internal Flow Examples

1) Engine oil is heated by flowing through a circular tube of diameter D= 50 mm and length L= 25 m and whose surface is maintained at 150 C.
   ( a) If the flow rate and inlet temperature of the oil are 0.5 kg/ s and 20 $^0$C, what is the outlet temperature $T_{mo}$? What is the total heat transfer rate q for the tube?
   ( b) For flow rates in the range $0.5 \le \dot{m} \le 2$ kg/s , compute and plot the variations of $T_{mo}$ and q with mass flow rate $\dot{m}$. For what flow rate( s) are q and $T_{mo}$ maximized? Explain your results.

**KNOWN:** Inlet temperature and flowrate of oil flowing through a tube of prescribed surface temperature and geometry.

**FIND:** (a) Oil outlet temperature and total heat transfer rate, and (b) Effect of flowrate.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Negligible temperature drop across tube wall, (2) Incompressible liquid with negligible viscous dissipation.

**PROPERTIES:** *Table A.5*, Engine oil (assume $T_{m,o}$ = 140°C, hence $\overline{T}_m$ = 80°C = 353 K): ρ = 852 kg/m$^3$, ν = 37.5 × 10$^{-6}$ m$^2$/s, k = 138 × 10$^{-3}$ W/m·K, Pr = 490, μ = ρ·ν = 0.032 kg/m·s, $c_p$ = 2131 J/kg·K.

**ANALYSIS:** (a) For constant surface temperature the oil outlet temperature may be obtained from Eq. 8.41b. Hence

$$T_{m,o} = T_S - \left(T_S - T_{m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{h}\right)$$

To determine $\overline{h}$, first calculate $Re_D$ from Eq. 8.6,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.5\,kg/s)}{\pi(0.05m)(0.032\,kg/m\cdot s)} = 398$$

Hence the flow is laminar. Moreover, from Eq. 8.23 the thermal entry length is

$$x_{fd,t} \approx 0.05 D\, Re_D\, Pr = 0.05(0.05\,m)(398)(490) = 486\,m\,.$$

Since L = 25 m the flow is far from being thermally fully developed. Since Pr > 5, $\bar{h}$ may be determined from Eq. 8.57

$$\overline{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{2/3}}\,.$$

With $Gz_D = (D/L) Re_D Pr = (0.05/25)398 \times 490 = 390$, it follows that

$$\overline{Nu}_D = 3.66 + \frac{26}{1+2.14} = 11.95.$$

Hence, $\bar{h} = \overline{Nu}_D \frac{k}{D} = 11.95 \frac{0.138\,W/m \cdot K}{0.05\,m} = 33\,W/m^2 \cdot K$ and it follows that

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$$T_{m,o} = 150^\circ C - \left(150^\circ C - 20^\circ C\right) \exp\left[-\frac{\pi(0.05\,m)(25\,m)}{0.5\,kg/s \times 2131\,J/kg \cdot K} \times 33\,W/m^2 \cdot K\right]$$

$T_{m,o} = 35°C$. <

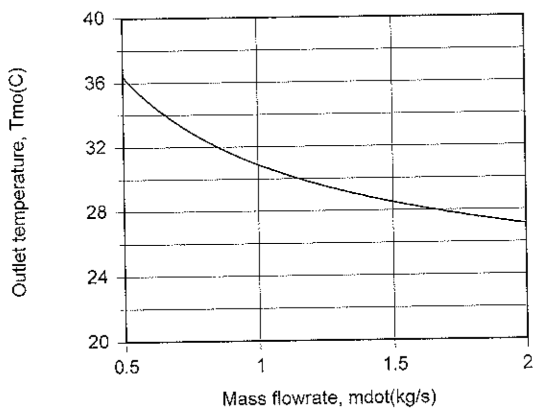
From the overall energy balance, Eq. 8.34, it follows that

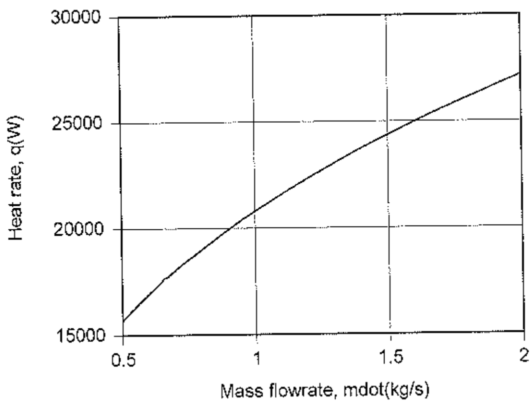
$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) = 0.5\,kg/s \times 2131\,J/kg \cdot K \times (35-20)^\circ C$$

q = 15,980 W. <

The value of $T_{m,o}$ has been grossly overestimated in evaluating the properties. The properties should be re-evaluated at $\bar{T} = (20 + 35)/2 = 27°C$ and the calculations repeated. Iteration should continue until satisfactory convergence is achieved between the calculated and assumed values of $T_{m,o}$. Following such a procedure, one would obtain $T_{m,o} = 36.4°C$, $Re_D = 27.8$, $\bar{h} = 32.8\,W/m^2 \cdot K$, and q = 15,660 W. The small effect of reevaluating the properties is attributed to the compensating effects on $Re_D$ (a large decrease) and Pr (a large increase).

(b) The effect of flowrate on $T_{m,o}$ and q was determined by using the appropriate IHT *Correlations* and *Properties* Toolpads.




The heat rate increases with increasing $\dot{m}$ due to the corresponding increase in $Re_D$ and hence $\bar{h}$. However, the increase is not proportional to $\dot{m}$, causing $\left(T_{m,o} - T_{m,i}\right) = q/\dot{m} c_p$, and hence $T_{m,o}$, to decrease with increasing $\dot{m}$. The maximum heat rate corresponds to the maximum flowrate ($\dot{m} = 0.20$ kg/s).

2) In the final stages of production, a pharmaceutical is sterilized by heating it from 25 $^0$C to 75 $^0$C as it moves at 0.2 m/ s through a straight thin-walled stainless steel tube of 12.7- mm diameter. A uniform heat flux is maintained by an electric resistance heater wrapped around the outer surface of the tube. If the tube is 10 m long, what is the required heat flux? If fluid enters the tube with a fully developed velocity profile and a uniform temperature profile, what is the surface temperature at the tube exit and at a distance of 0.5 m from the entrance? Fluid properties may be approximated as

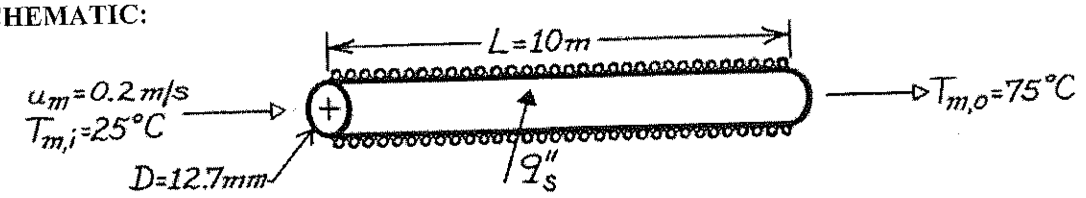
$\rho = 1000\,kg/m^3, c_p = 4000\,J/kg.K, \mu = 2\times10^{-3}\,kg/m.s$

$k = 0.8\,W/m.K, Pr = 10.$

**KNOWN:** Inlet and outlet temperatures and velocity of fluid flow in tube. Tube diameter and length.

**FIND:** Surface heat flux and temperatures at x = 0.5 and 10 m.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to surroundings, (4) Incompressible liquid with negligible viscous dissipation, (5) Negligible axial conduction.

**PROPERTIES:** Pharmaceutical (given): $\rho$ = 1000 kg/m$^3$, $c_p$ = 4000 J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, k = 0.80 W/m·K, Pr = 10.

**ANALYSIS:** With

$$\dot{m} = \rho VA = 1000\ \text{kg/m}^3(0.2\ \text{m/s})\pi(0.0127\ \text{m})^2/4 = 0.0253\ \text{kg/s}$$

Eq. 8.34 yields

$$q = \dot{m}\, c_p\left(T_{m,o} - T_{m,i}\right) = 0.0253\ \text{kg/s}\left(4000\ \text{J/kg}\cdot\text{K}\right)50\ \text{K} = 5060\ \text{W}.$$

The required heat flux is then

$$q''_s = q/A_s = 5060\ \text{W}/\pi(0.0127\ \text{m})10\ \text{m} = 12{,}682\ \text{W/m}^2. \qquad <$$

With

$$Re_D = \rho VD/\mu = 1000\ \text{kg/m}^3(0.2\ \text{m/s})0.0127\ \text{m}/2\times10^{-3}\ \text{kg/s}\cdot\text{m} = 1270$$

the flow is laminar and Eq. 8.23 yields

$$x_{fd,t} = 0.05 Re_D\, Pr\, D = 0.05(1270)10(0.0127\ \text{m}) = 8.06\ \text{m}.$$

Hence, with fully developed hydrodynamic and thermal conditions at x = 10 m, Eq. 8.53 yields

$$h(10\ \text{m}) = Nu_{D,fd}(k/D) = 4.36(0.80\ \text{W/m}\cdot\text{K}/0.0127\ \text{m}) = 274.6\ \text{W/m}^2\cdot\text{K}.$$

Hence, from Newton's law of cooling,

$$T_{s,o} = T_{m,o} + \left(q_s'' / h\right) = 75^\circ C + \left(12{,}682\ W/m^2 / 274.6\ W/m^2 \cdot K\right) = 121^\circ C.$$ <

At x = 0.5 m, $(x/D)/(Re_D Pr) = 0.0031$ and Figure 8.10 yields $Nu_D \approx 8$ for a thermal entry region with uniform surface heat flux. Hence, h(0.5 m) = 503.9 $W/m^2 \cdot K$ and, since $T_m$ increases linearly with x, $T_m(x = 0.5\ m) = T_{m,i} + (T_{m,o} - T_{m,i})\ (x/L) = 27.5^\circ C$. It follows that

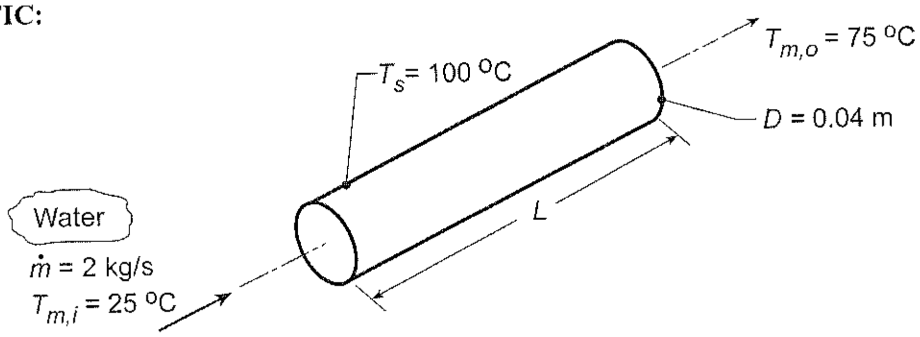
$$T_s\left(x = 0.5\ m\right) \approx 27.5^\circ C + \left(12{,}682\ W/m^2 / 503.9\ W/m^2 \cdot K\right) = 52.7^\circ C.$$ <

3) Water flowing at 2 kg/ s through a 40- mm- diameter tube is to be heated from 25 to 75 $^0$C by maintaining the tube surface temperature at 100 $^0$C. What is the required tube length for these conditions?

**KNOWN:** Flow rate, inlet temperature and desired outlet temperature of water passing through a tube of prescribed diameter and surface temperature.

**FIND:** (a) Required tube length, L, for prescribed conditions, (b) Required length using tube diameters over the range $30 \le D \le 50$ mm with flow rates $\dot{m}$ = 1, 2 and 3 kg/s; represent this design information graphically, and (c) Pressure gradient as a function of tube diameter for the three flow rates assuming the tube wall is smooth.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Incompressible liquid with negligible viscous dissipation, (3) Constant properties.

**PROPERTIES:** *Table A.6*, Water ($\bar{T}_m$ = 323 K): $c_p$ = 4181 J/kg·K, $\mu = 547 \times 10^{-6}$ N·s/m$^2$, k = 0.643 W/m·K, Pr = 3.56.

**ANALYSIS:** (a) From Eq. 8.6, the Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 2\,\text{kg/s}}{\pi(0.04\,\text{m})547\times10^{-6}\,\text{N}\cdot\text{s/m}^2} = 1.16\times10^5. \quad (1)$$

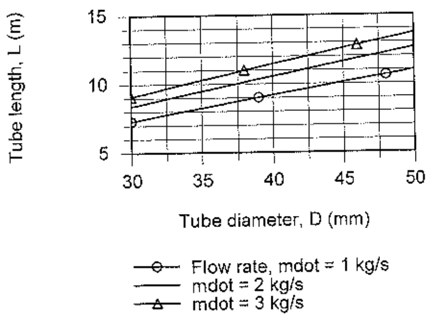
Hence the flow is turbulent, and assuming fully developed conditions throughout the tube, it follows from the Dittus-Boelter correlation, Eq. 8.60,

$$\bar{h} = \frac{k}{D}0.023\,Re_D^{4/5}\,Pr^{0.4} = \frac{0.643\,\text{W/m}\cdot\text{K}}{0.04\,\text{m}}0.023\left(1.16\times10^5\right)^{4/5}(3.56)^{0.4} = 6919\,\text{W/m}^2\cdot\text{K} \quad (2)$$

From Eq. 8.41a, we then obtain

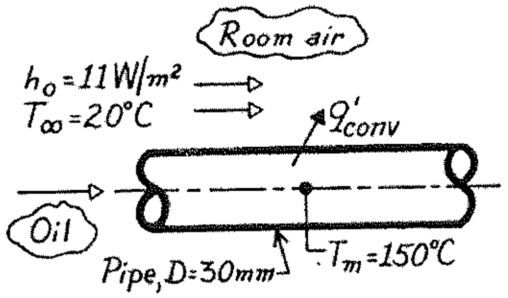
$$L = \frac{-\dot{m}c_p \ell n(\Delta T_o/\Delta T_i)}{\pi D \bar{h}} = -\frac{2\,\text{kg/s}(4181\,\text{J/kg}\cdot\text{K})\ell n\left(25^\circ\text{C}/75^\circ\text{C}\right)}{\pi(0.04\,\text{m})6919\,\text{W/m}^2\cdot\text{K}} = 10.6\,\text{m}.$$ <

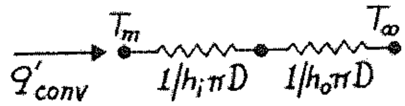
(b) Using the *IHT Correlations Tool, Internal Flow*, for fully developed *Turbulent Flow*, along with appropriate energy balance and rate equations, the required length L as a function of flow rate is computed and plotted on the right.



4) Engine Oil with a mean temperature of 147 $^0$C flows slowly at 0.00578 m/s through a 20 m long, thin- walled pipe of 30- mm inner diameter. The pipe is suspended in a room for which the air temperature is 20 $^0$C and the convection coefficient at the outer tube surface is 11 W/ m$^2$ K. Estimate the heat loss per unit length of tube.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Fully developed flow, (4) Radiation exchange between pipe and room negligible.

**PROPERTIES:** *Table A-5*, Unused engine oil ($T_m$ = 150°C = 423K): k = 0.133 W/m·K.

$\nu = 6.94 \times 10^{-6}$ m$^2$/s
$Pr = 103$

**ANALYSIS:** The rate equation, for a unit length of the pipe, can be written as

$$q'_{conv} = \frac{(T_m - T_\infty)}{R'_t}$$

where the thermal resistance is comprised of two elements,

$$R'_t = \frac{1}{h_i \pi D} + \frac{1}{h_o \pi D} = \frac{1}{\pi D}\left(\frac{1}{h_i} + \frac{1}{h_o}\right).$$

$$Re_D = \frac{U_m D}{\nu} = \frac{(0.00578)(30/100)}{6.94 \times 10^{-6}} \simeq 25$$

$$X_{fdh} = 0.05\, Re_D D = (0.05)(25)(30/100) = 0.037 \text{ m}$$

$$x_{fdt} = 0.05\, Re_D D\, Pr = (0.05)(25)(100)(30/100) = 3.75 \text{ m}$$

flow is hydrodynamically and thermally fully developed.

$$Nu_D = \frac{h_i D}{K} = 3.66 \Rightarrow h_i = 3.66)\left(\frac{0.133}{0.03}\right) = 16.2 \frac{W}{m^2 K}$$

$$q' = \frac{(150-20)}{\frac{1}{\pi (0.03)}\left[\frac{1}{16.2} + \frac{1}{11}\right]} = 80.3 \text{ W/m}$$

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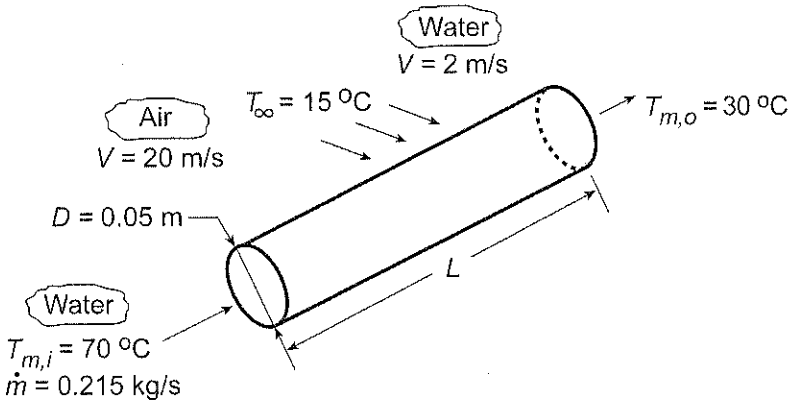
5) Water at a flow rate of $\dot{m} = 0.215$ kg/ s is cooled from 70 $^0$C to 30 $^0$C by passing it through a thin- walled tube of diameter D= 50 mm and maintaining a coolant at T =15 $^0$C in cross flow over the tube.

( a) What is the required tube length if the coolant is air and its velocity is V= 20 m/ s?

( b) What is the tube length if the coolant is water and V= 2 m/ s?

**FIND:** (a) Required tube length for air in cross flow at prescribed velocity, (b) Required tube length for water in cross flow at a prescribed velocity.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Constant properties, (3) Negligible tube wall conduction resistance, (4)Water is incompressible liquid with negligible viscous dissipation.

**PROPERTIES:** *Table A.6*, water ($\overline{T}_m$ = 50°C = 323 K): $c_p$ = 4181 J/kg·K, $\mu = 548 \times 10^{-6}$ N·s/m$^2$, k = 0.643 W/m·K, Pr = 3.56. *Table A.4*, air (assume $T_f$ = 300 K): $\nu = 15.89 \times 10^{-6}$ m$^2$/s, k = 0.0263 W/m·K, Pr = 0.707. *Table A.6*, water (assume $T_f$ = 300 K): $\nu = 0.858 \times 10^{-6}$ m$^2$/s, k = 0.613 W/m·K, Pr = 5.83.

**ANALYSIS:** The required heat rate may be determined from the overall energy balance,

$$q = \dot{m}c_p\left(T_{m,i} - T_{m,o}\right) = 0.215\,\text{kg/s}\left(4181\,\text{J/kg}\cdot\text{K}\right)40^\circ\text{C} = 35{,}960\,\text{W}$$

and the required tube length may be determined from the rate equation, Eq. 8.46a,

$$L = \frac{q}{U\pi D\Delta T_{\ell m}}$$

where

$$\Delta T_{\ell m} = \frac{\left(T_{m,i} - T_\infty\right) - \left(T_{m,o} - T_\infty\right)}{\ell n\left(\dfrac{T_{m,i} - T_\infty}{T_{m,o} - T_\infty}\right)} = 30.8^\circ\text{C} \qquad \text{and} \qquad 1/U = 1/h_i + 1/h_o.$$

With

$$Re_{D_i} = 4\dot{m}/\pi D\mu = 0.860\,\text{kg/s}\Big/\pi\left(0.05\,\text{m}\right)548\times10^{-6}\,\text{N}\cdot\text{s/m}^2 = 9991$$

the flow is turbulent and, assuming fully developed flow throughout the tube, the inside convection coefficient is determined from Eq. 8.62

$$Nu_{D_i} = \frac{(f/8)(Re_{D_i} - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} = \frac{(0.0315/8)(9991 - 1000)3.56}{1 + 12.7(0.0315/8)^{1/2}(3.56^{2/3} - 1)} = 61.1$$

where f = (0.79 $\ln Re_{Di}$ = 1.64)$^{-2}$ = 0.0315

$$h_i = Nu_{D_i}\,k/D = 61.1\left(0.643\,\text{W/m}\cdot\text{K}\right)/0.05\,\text{m} = 786\,\text{W/m}^2\cdot\text{K}$$

(a) For air in cross flow at 20 m/s, $Re_{D_o} = VD/\nu = 20$ m/s(0.05 m)/15.89 × $10^{-6}$ m²/s = 62,933. From the Churchill/Bernstein correlation, it follows that

$$Nu_{D_o} = 0.3 + \frac{0.62\, Re_{D_o}^{1/2}\, Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_{D_o}}{282,000}\right)^{5/8}\right]^{4/5} = 158.7$$

$$h_o = Nu_{D_o}\, k/D = 158.7\left(0.0263\, W/m\cdot K\right)/0.05\, m = 83.5\, W/m^2\cdot K$$

Hence, $U = \left(1/h_i + 1/h_o\right)^{-1} = 75.5\ W/m^2\cdot K$ and

$$L = \frac{35,960\, W}{\left(75.5\, W/m^2\cdot K\right)\pi\left(0.05\, m\right)30.8^\circ C} = 98.5\, m$$ <

(b) For water in cross flow at 2 m/s, $Re_{D_o} = 2$ m/s(0.05 m)/0.858 × $10^{-6}$ m²/s = 116,550, and the correlation yields $Nu_{D_o} = 527.3$. Hence,

$$h_o = Nu_{D_o}\, k/D = 527.3\left(0.613\, W/m\cdot K\right)/0.05\, m = 6,465\, W/m^2\cdot K$$

$$U = \left(1/h_i + 1/h_o\right)^{-1} = 701\ W/m^2\cdot K$$

Hence,

$$L = \frac{35,960\, W}{\left(701\, W/m^2\cdot K\right)\pi\left(0.05\, m\right)30.8^\circ C} = 10.6\, m$$ <

**COMMENTS:** The foregoing results clearly indicate the superiority of water (relative to air) as a heat transfer fluid. Note the dominant contribution made by the smaller convection coefficient to the value of U in each of the two cases.