Chapter-3
Examples

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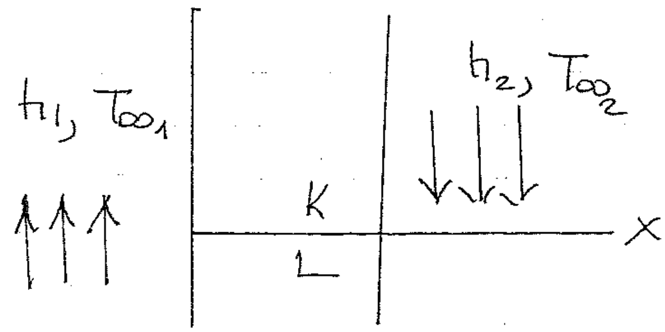
Example - 1)

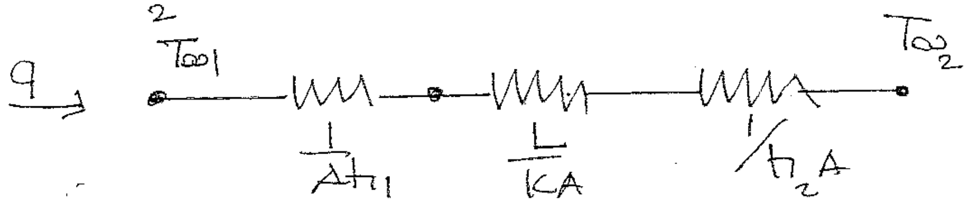
Consider a slab of thickness L as illustrated in figure below. A fluid



at temperature $T_{\infty 1}$ with heat transfer coefficient h_1 flows over the surface at $x=0$, and another fluid at temperature $T_{\infty 2}$ with a heat transfer coefficient h_2 flows over surface at $x=L$ of plate.

Find the heat flow through area A.

$A = 1\ m^2$, $T_{\infty 1} = 130°C$, $h_1 = 250\ W/m^2°C$, $h_2 = 500\ W/m^2°C$, $L = 4\ cm$, $k = 20\ W/m°C$, $T_{\infty 2} = 30°C$.



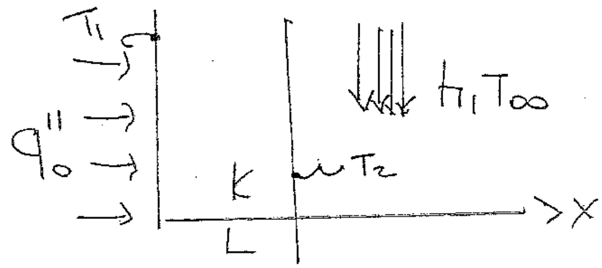
$$q = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{Ah_1} + \frac{L}{KA} + \frac{1}{Ah_2}}$$

$$R_{tot} = \frac{1}{Ah_1} + \frac{L}{KA} + \frac{1}{Ah_2} = 8 \times 10^{-3}$$

$$q = \frac{130-30}{8 \times 10^{-3}} = 12.5\ KW$$

Example - 2

An iron plate of thickness L with thermal conductivity k is subjected to a constant uniform heat flux q_o'' (W/m²) at the boundary surface $x=0$. From the other boundary surface $x=L$ heat is dissipated by convection into a fluid at T_∞ with heat transfer coefficient h.



Develop expressions for the determination of surface temperatures T_1 and T_2 at the surfaces $x=0$ and $x=L$ respectively. Calculate T_1 and T_2 for $L = 2\,cm$, $k = 20\,W/m$, $q_o'' = 10^5\,W/m^2$, $T_\infty = 50°C$, $h = 5000\,W/m^2°C$.

$$Aq_o'' \rightarrow \overset{T_1}{\circ} \underset{\frac{L}{Ak}}{\text{—WW—}} \overset{T_2}{\circ} \underset{1/Ah}{\text{—WW—}} \overset{T_\infty}{\circ}$$

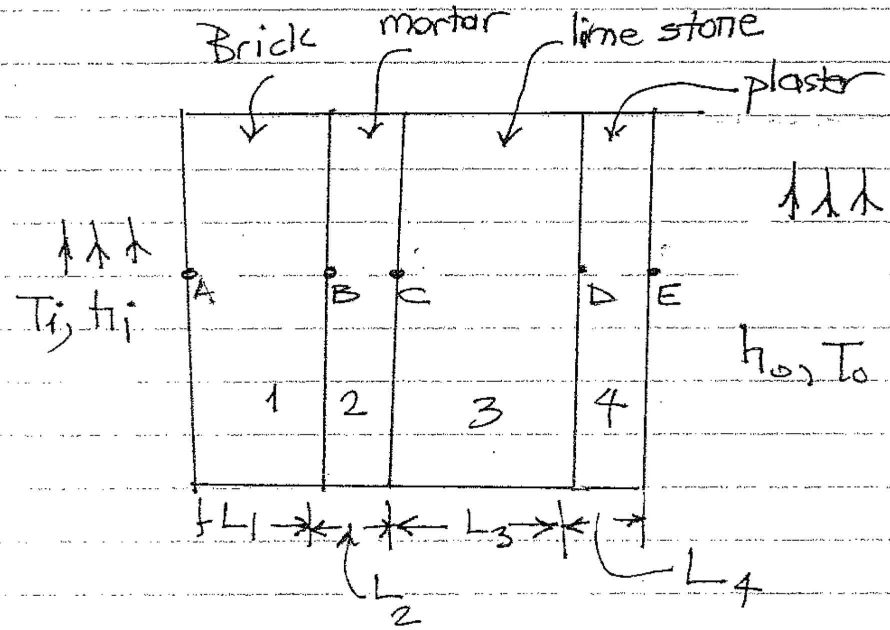
$$Aq_o'' = \frac{T_1 - T_2}{\frac{L}{Ak}} = \frac{T_2 - T_\infty}{\frac{1}{Ah}} = \frac{T_1 - T_\infty}{\frac{L}{Ak} + \frac{1}{Ah}}$$

$$T_1 = \left(\frac{L}{k} + \frac{1}{h}\right) q_o'' + T_\infty \qquad = 350°C$$

$$T_2 = \frac{q_o''}{h} + T_\infty \qquad = 250°C$$

Example -3

A wall is constructed by several layers.



$L_1 = 0.25m$ $\quad$ $k_1 = 0.66 \; W/mK$

$L_2 = 0.025m$ $\quad$ $k_2 = 0.7 \; W/mK$

$L_3 = 0.1m$ $\quad$ $k_3 = 0.66 \; W/mK$

$L_4 = 0.0125m$ $\quad$ $k_4 = 0.7 \; W/mK$

$h_i = 5.8 \; W/m^2K$ $\quad$ $h_o = 11.6 \; W/m^2K$

$T_i = 26°C$ $\quad$ $T_o = -7°C$

Find

a) overall heat transfer coefficient

b) overall thermal resistance

c) heat flow rate through composite wall

d) temperature at the inter face of mortar and limestone

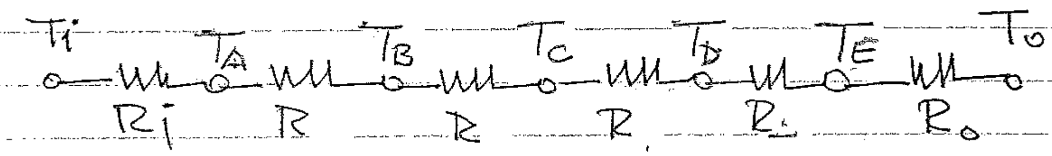
solution

Assume:

1- steady state heat conduction

2) one-dimensional heat conduction
3) constant properties

R_1



$$R_i = \frac{1}{Ah_i} = 0.1724 \text{ K/W}$$

$$R_1 = \frac{L_1}{Ak_1} = 0.378 \text{ K/W}$$

$$R_2 = \frac{L_2}{Ak_2} = 0.0357 \text{ K/W}$$

$$R_3 = \frac{L_3}{Ak_3} = 0.1515 \text{ K/W}$$

$$R_4 = \frac{L_4}{Ak_4} = 0.0178 \text{ K/W}$$

$$R_o = \frac{1}{Ah_o} = 0.0862 \text{ K/W}$$

a) $$= \frac{1}{A\sum R} = \frac{1}{A(R_i + R_1 + R_2 + R_3 + R_4 + R_o)}$$

$$= \frac{1}{0.8424} = 1.187 \text{ W/m}^2\text{K}$$

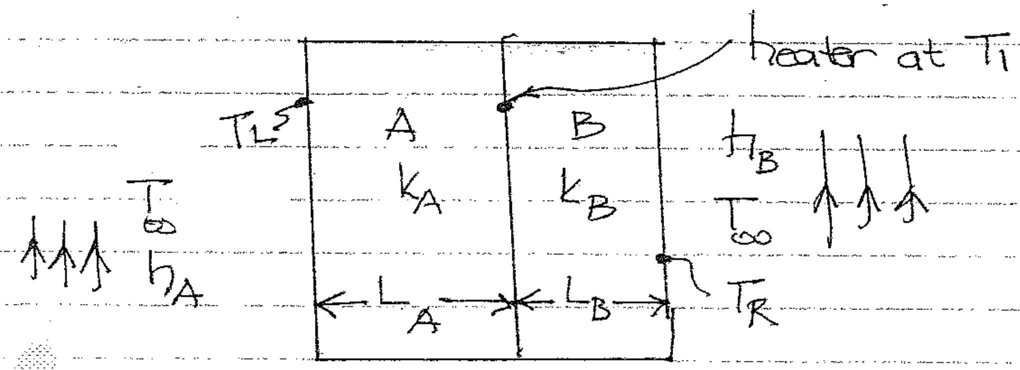
b) $$\sum R = \frac{1}{UA} = \frac{1}{(1.187)(1)} = 0.8424 \text{ K/W}$$

c) $$Q = \frac{T_i - T_o}{\sum R} = \frac{26 - (-7)}{0.8424} = 39.17 \text{ W/m}^2$$

d) $$Q = \frac{T_i - T_c}{R_i + R_1 + R_2} \Rightarrow T_c = T_i - 39.17(0.1724 + 0.378 + 0.0357)$$
$$= 3°C$$

Example - 4

A square plate heater (15 cm x 15 cm) is inserted between two slabs.



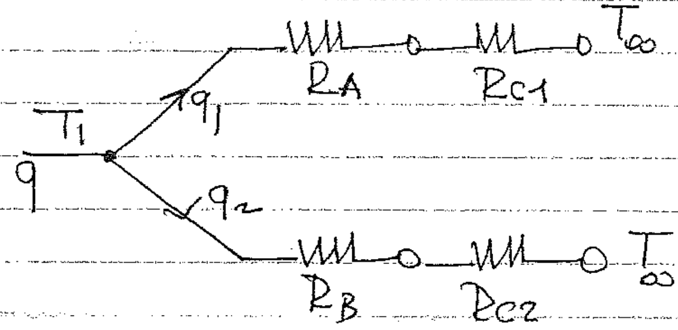
$L_A = 2\,cm$
$k_A = 50\ W/mK$
$h_A = 200\ W/mK$
$T_\infty = 25°C$

$L_B = 1\,cm$
$k_B = 0.2\ W/mK$
$h_B = 50\ W/m^2K$
$Q = 1\,kW = 1000\,W$

$A = 15\,cm \times 15\,cm$
$= 225 \times 10^{-4}\,m^2$

Find
a) maximum temperature of the system
b) outer surface temperatures on both slabs

Assume
1- steady state
2- one-dimensional heat flow
3) constant properties



$$R_A = \frac{L_A}{A K_A} = 0.0177\ K/W \qquad R_B = \frac{L_B}{A K_B} = 2,22\ K/W$$

$$R_{c1} = \frac{1}{A h_A} = 0.22\ K/W \qquad R_{c2} = \frac{1}{A h_B} = 0.88\ K/W$$

$$q = q_1 + q_2$$

$$q = \frac{(T_1 - T_\infty)}{R_A + R_{c1}} + \frac{(T_1 - T_\infty)}{R_B + R_{c2}}$$

$$1000 = \frac{T_1 - 25}{0.0177 + 0.22} + \frac{T_1 - 25}{2,22 + 0,88}$$

$$T_1 = 245,77°C$$

b) $$q_1 = \frac{T_1 - T_\infty}{R_A + R_{c1}} = \frac{245,77 - 25}{0.2377} = 928.77\ W$$

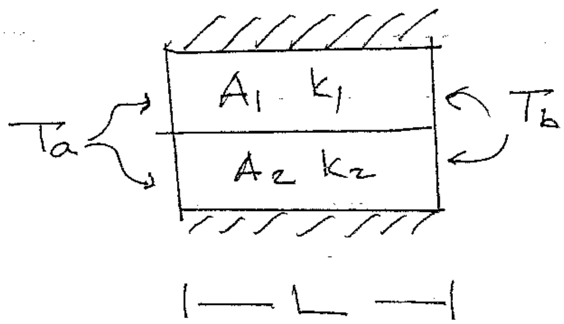
$$q_1 = \frac{T_1 - T_L}{R_A} \Rightarrow T_L = 229,33°C$$

c) $$q_2 = \frac{T_1 - T_\infty}{R_B + R_{c2}} = \frac{245,77 - 25}{2,22 + 0,88} = 71.21\ W$$

$$q_2 = \frac{T_1 - T_R}{R_B} \Rightarrow T_R = 87.6°C$$

Example - 5

Consider the composite of two materials combined in parallel paths with the ends maintained at uniform temperatures as illustrated in the figure below.



Various quantities are specified.

$A_1 = 0.2\ m^2$

$k_1 = 20$ W/m K

$A_2 = 0.4\ m^2$

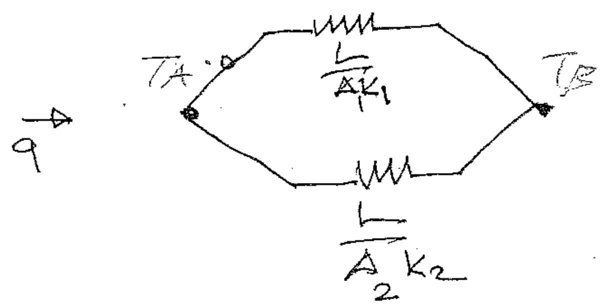
$k_2 = 15$ W/m °C

$L = 0.15$ m $\quad T_a = 150$ °C,

$T_b = 30$ °C

Calculate the rate of heat transfer q across composite material.

0.5m



$$\frac{1}{R} = \frac{A_1 k_1}{L} + \frac{A_2 k_2}{L}$$

$$\frac{1}{R} = 20$$

$$q = \frac{T_a - T_b}{R} = \frac{150-30}{1/20}$$

$$= 2400 \text{ W}$$

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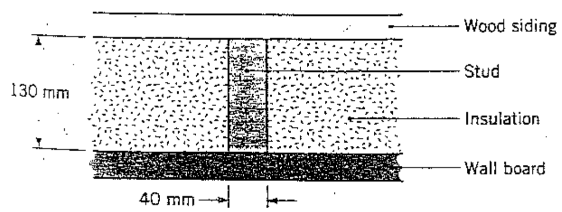
Example- 6

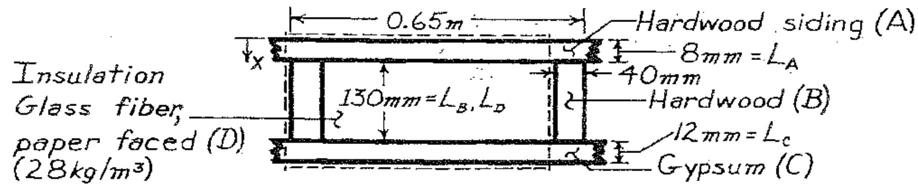
Consider a composite wall that includes an 8 mm thick hardwood siding, 40 mm by 130 mm hardwood studs on 0.65 m centers with glass fiber insulation (paper faced, 28 kg/m^3), and a 12 mm layer of gypsum (vermiculite) wall board. What is the thermal resistance associated with a wall that is 2.5 m high by 6.5 m wide (having 10 studs, each 2.5 m high)?



FIND: Wall thermal resistance.

2.5m X6.5m
10 studs
each stud
2.5m high

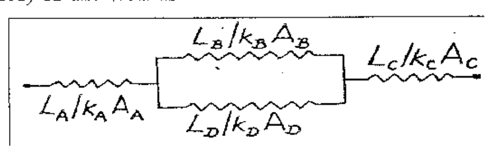
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of composite depends only on x (surfaces normal to x are isothermal), (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: *Table A-3* ($T \approx 300K$): Hardwood siding, $k_A = 0.094$ W/m·K; Hardwood, $k_B = 0.16$ W/m·K; Gypsum, $k_C = 0.17$ W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), $k_D = 0.038$ W/m·K.

ANALYSIS: Using the isothermal surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is



$$(L_A / k_A A_A) = \frac{0.008\text{m}}{0.094 \text{ W/m}\cdot\text{K } (0.65\text{m}\times 2.5\text{m})} = 0.0524 \text{ K/W}$$

$$(L_B / k_B A_B) = \frac{0.13\text{m}}{0.16 \text{ W/m}\cdot\text{K } (0.04\text{m}\times 2.5\text{m})} = 8.125 \text{ K/W}$$

$$(L_D / k_D A_D) = \frac{0.13\text{m}}{0.038 \text{ W/m}\cdot\text{K} (0.61\text{m}\times 2.5\text{m})} = 2.243 \text{ K/W}$$

$$(L_C / k_C A_C) = \frac{0.012\text{m}}{0.17 \text{ W/m}\cdot\text{K} (0.65\text{m}\times 2.5\text{m})} = 0.0434 \text{ K/W}.$$

The equivalent resistance of the core is

$$R_{eq} = (1/R_B + 1/R_D)^{-1} = (1/8.125 + 1/2.243)^{-1} = 1.758 \text{ K/W}$$

and the total unit resistance is

$$R_{tot,1} = R_A + R_{eq} + R_C = 1.854 \text{ K/W}.$$

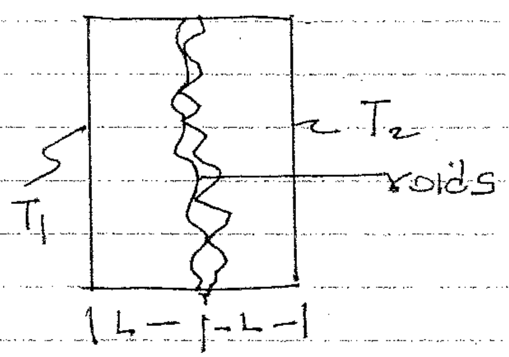
With 10 such units in parallel, the total wall resistance is

$$R_{tot} = \left(10 \times 1/R_{tot,1}\right)^{-1} = 0.1854 \text{ K/W}.$$

Example - 7

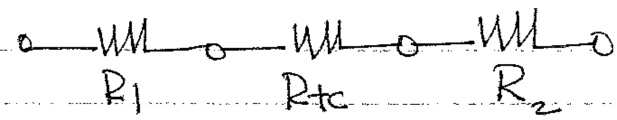
Two large aluminium plates ($k = 240$ W/mK) each 1 cm thick with 10 μm surface roughness are placed in contact under a pressure of 1 bar in air ($k = 0.026$ $\frac{W}{mK}$). The temperature at inside and outside surfaces are 400 °C, and 150 °C.
Calculate
(a) heat flux
(b) temperature drop due to contact resistance

solution



$k = 240$ W/mK
$L_a = 10\ \mu m = 10 \times 10^{-6}$ m
$T_2 = 150$ °C
$L = 0.01$ m
$T_1 = 400$ °C
$k_a = 0.026$ W/mK

thermal resistances per 1 m².



$$R_1 = \frac{L}{AK} = \frac{0.01}{(1)(240)} = 4.167 \times 10^{-5}\ \frac{m^2 K}{W}$$

$$R_3 = \frac{L}{AK} = 4.167 \times 10^{-5}\ \frac{m^2 K}{W}$$

$$R_{tc} = 2.75 \times 10^{-4}\ m^2 K/W$$

$$q = \frac{T_1 - T_2}{\sum R} = \frac{T_1 - T_2}{R_1 + R_2 + R_3}$$

$$= \frac{400 - 150}{4.167 \times 10^{-5} + 2.75 \times 10^{-4} + 4.167 \times 10^{-5}}$$

$$= 2.79 \times 10^{4} \ W/m^2$$

b) temperature drop across contact resistance

$$q = \frac{\Delta T_c}{R_{tc}} \qquad \Delta T_c = q \, R_{tc}$$

$$= (2.79 \times 10^{4})(2.75 \times 10^{-4}) = 7.67 \, ^\circ C$$

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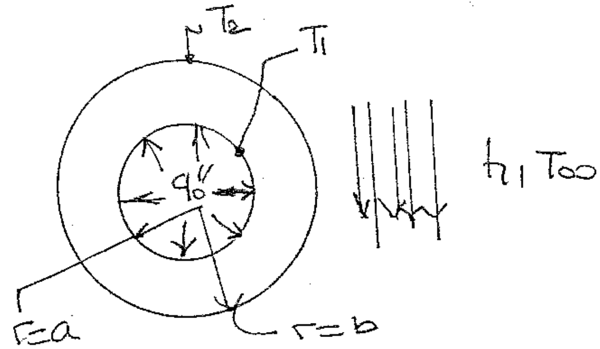
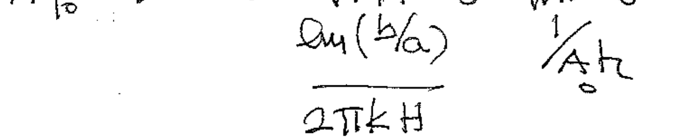
Example - 8

A hollow cylinder with inner cylinder radius $r = a$ and outer radius $r = b$ is heated at the inner surface at a rate of q_0'' (W/m^2), and dissipates heat by convection from the outer surface into a fluid at temperature T_∞ with a heat transfer coefficient h. There is no



energy generation and thermal conductivity constant. Develop expressions for the determination of temperatures at T_1 and T_2 of inner and outer surfaces of the cylinder. Calculate T_1 and T_2 for $a = 3$ cm, $b = 5$ cm, $h = 400$ $W/m^2 °C$, $T_\infty = 100°C$, $k = 15$ W/mK, $q_0'' = 10^5$ W/m^2



$$(2\pi a)(H)q_0'' = \frac{T_1 - T_2}{\frac{\ln(b/a)}{2\pi k H}} = \frac{T_2 - T_\infty}{\frac{1}{2\pi b H h}}$$

H - cylinder length

$$= \frac{T_1 - T_\infty}{\dfrac{\ln(b/a)}{2\pi k H} + \dfrac{1}{2\pi b H h}}$$

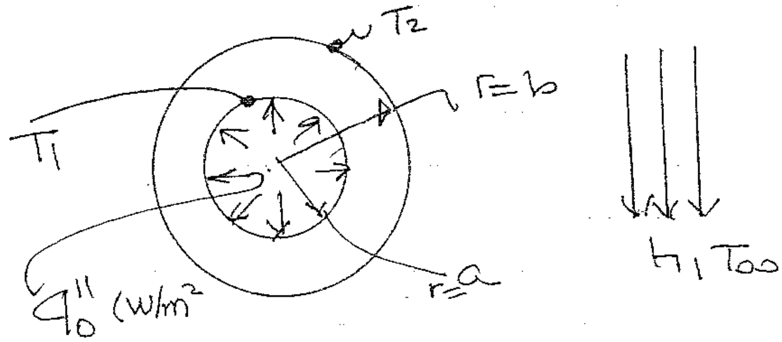
$\therefore$

$$T_1 = \left[\frac{a}{k} \ln(b/a) + \frac{a}{bh} \right] q_0'' + T_\infty = 352.2\ ^\circ C$$

$$T_2 = \left(\frac{a}{bh}\right) q_0'' + T_\infty = 250\ ^\circ C$$

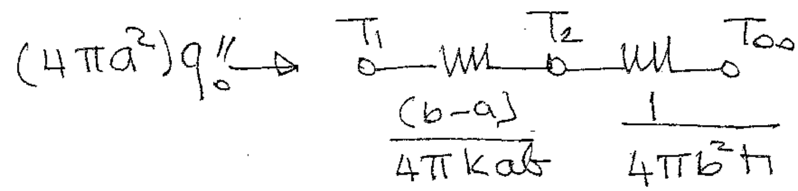
Example - 9

A hollow sphere of inside radius $r=a$ and outside radius $r=b$ is electrically heated at the inner surface at a constant rate of q_0'' (W/m^2). At the outer surface it dissipates heat by convection into fluid at temperature T_∞ with a heat transfer coefficient h. Thermal conductivity of solid is constant.



Develop expressions for the determination of the inner and outer surface temperatures T_1 and T_2 of sphere.

Calculate the inner and outer surface temperatures for $a = 3$ cm, $b = 5$ cm, $h = 400\ W/m^2\,°C$, $T_\infty = 100\,°C$, $k = 15\ W/m\,°C$ and $q_0'' = 10^5\ W/m^2$.



$$4\pi a^2 q_0'' = \frac{T_1 - T_2}{\frac{b-a}{4\pi kab}} = \frac{T_2 - T_\infty}{4\pi b^2 h} = \frac{T_1 - T_\infty}{\frac{b-a}{4\pi kab} + \frac{1}{4\pi b^2 h}}$$

$$T_1 = \left[\frac{a(b-a)}{bk} + \left(\frac{a}{b}\right)^2 \frac{1}{h}\right] q_0'' + T_\infty = 270\,°C$$

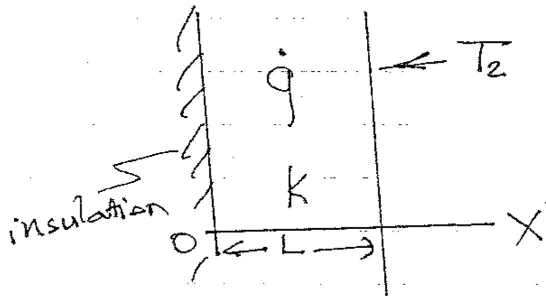
$$T_2 = \left(\frac{a}{b}\right)^2 \frac{q_0''}{h} + T_\infty = 190\,°C$$

Example -10

A pressure vessel for a nuclear reactor is approximated as a large flat plate of thickness L. The inside surface of the plate at $x=0$ is insulated, the outside surface at $x=L$ is maintained at a uniform temperature T_2, and the gamma-ray heating of the plate can be represented as a heat generation term in the form

$$\dot{q}(x) = q_o \bar{e}^{\gamma x} \quad W/m^3$$

where q_o and γ are constants



a) develop an expression for temperatur distribution in the plate

b) develop an expression for the temperature at $x=0$ of the plate

c) develop an expression for heat flux at the outer surface

Solution

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$x=0 \quad \frac{dT}{dx} = 0$$

$$x=L \quad T = T_2$$

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$$\frac{d^2T}{dx^2} = -\frac{g_o}{k}\, e^{-\gamma x}$$

first integration

$$\frac{dT}{dx} = \frac{g_o}{k\gamma}\, e^{-\gamma x} + C_1$$

$$x = 0 \quad \frac{dT}{dx} = 0 \implies C_1 = -\frac{g_o}{k\gamma}$$

second integration

$$T(x) = -\frac{g_o}{k\gamma^2}\, e^{-\gamma x} + C_1 x + C_2$$

$$x = L \quad T = T_2 \implies C_2 = T_2 + \frac{g_o}{k\gamma^2}\, e^{-\gamma L} + \frac{g_o L}{k\gamma}$$

$$T(x) = \frac{g_o}{k\gamma^2}\left[e^{-\gamma L} - e^{-\gamma x}\right] + \frac{g_o L}{k\gamma}\left(1 - \frac{x}{L}\right) + T_2$$

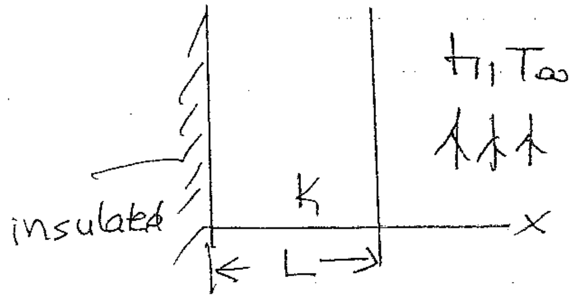
b) $T(0) = \frac{g_o}{k\gamma^2}\left[e^{-\gamma L} - 1\right] + \frac{g_o L}{k\gamma} + T_2$

b) heat flux at $x = L$

$$q = -k\frac{dT}{dx}\bigg|_{x=L} = \frac{g_o}{\gamma}\left(1 - e^{-\gamma L}\right)$$

Example - 11

Consider a slab of thickness L and constant thermal conductivity k in which energy is generated at a constant rate of $\dot{q}$ (W/m^3). The boundary



at $x=0$ is insulated and that $x=L$ dissipates heat by convection with a heat transfer coefficient h into a fluid at a temperature T_∞. Develop expressions for temperature $T(x)$ and heat flux $q''(x)$ in the slab. Calculate temperatures at $x=0$ and $x=L$ under the following conditions.

$L = 1$ cm, $k = 20$ W/mK, $\dot{q} = 8\times10^7$ W/m^3

$h = 4000$ W/m°C, $T_\infty = 100$°C.

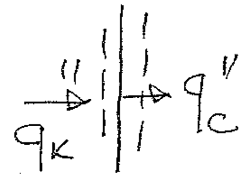
Solution

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \qquad 0<x<L$$

$$x=0 \qquad \frac{dT}{dx} = 0$$

$$x=L \qquad k\frac{dT}{dx} + hT = hT_\infty$$

i.e



$$q_k'' = q_c''$$

$$-k\frac{dT}{dx} = h[T - T_\infty]$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + c_1 \quad \therefore \quad x=0 \quad \frac{dT}{dx}=0$$

$$c_1 = 0$$

$$T = -\frac{\dot{q}}{2k}x^2 + c_2$$

using B.C at $x=L$

$$-\dot{q}L + h\left[-\frac{\dot{q}L^2}{2k} + c_2\right] = hT_\infty$$

$$\therefore \quad c_2 = \frac{\dot{q}L^2}{2k} + \frac{\dot{q}L}{h} + T_\infty$$

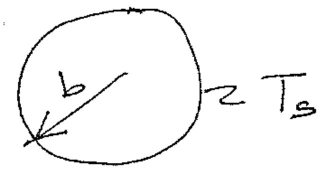
so $$T(x) = \frac{\dot{q}L^2}{2k}\left[1 - \left(x/L\right)^2\right] + \frac{\dot{q}L^2}{h} + T_\infty$$

and heat flux

$$q''(x) = -k\frac{dT}{dx} = \dot{q}x$$

$T(0) = 500°C$
$T(L) = 300°C$

Example - 12
Consider a solid cylinder of radius $r=b$ in which energy is generated at a constant rate of g_o (W/m^3), while the boundary surface at $r=b$ is maintained at constant temperature T_s



Develop an expression for one-dimensional radial, steady-state temperature distribution $T(r)$ and heat flux $q(r)$
Calculate the center temperature $T(0)$ and the heat flux at the boundary surface $r=b$ for $b=1$ cm $g_o = 2\times10^8$ W/m³, $k=20$ W/m °C, $T_s = 100$ °C.

Solution

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{g_o}{k}=0 \qquad 0<r<b$$

$$r=0 \qquad \frac{dT}{dr}=0$$

$$r=b \qquad T=T_s$$

$$\frac{dT}{dr}=-\frac{g_o}{2k}r+C_1/r$$

$$T(r)=-\frac{g_o}{4k}r^2+C_1\ln r+C_2$$

using B.C.s

$$r=0 \quad dT/dr=0 \longrightarrow C_1=0$$

$$r=b \quad T=T_s \longrightarrow C_2=\frac{g_o b^2}{4k}+T_s$$

$$T = \frac{g_o b^2}{4k}\left[1 - r^2/b^2\right] + T_s$$

$$T(0) = T_s + \frac{g_o b^2}{4k} \quad \text{center temperature}$$

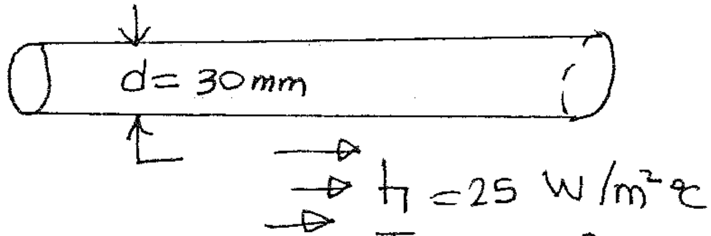
$$q_r = -k \left.\frac{dT}{dr}\right|_{r=b} = \frac{g_o b}{2}$$

b)
$$T(0) = \frac{2\times10^8 \times (0.01)^2}{4\times20} + 100 = 350\ ^\circ c$$

$$q'' = \frac{g_o b}{2} = (2\times10^8 \times 0.01)/2 = 10^6\ w/m^2$$

Example - 13

250 A current flows through a copper cable (as shown in the figure) exposed to convective air.



a) Find maximum temperature in the wire
b) Find surface of " "

Assume 1- Dimensional conduction
steady state
constant properties
uniform heat generation

for copper at T = 300 K
$k = 401$ W/mK

$$T_s = T_\infty + \frac{\dot{q}\, r_o}{2h}$$

$$\dot{q} = \frac{E_g}{\forall} = \frac{I^2 R_e}{\left(\frac{\pi d^2}{4}\right) L} = \frac{I^2 R_e'}{\frac{\pi d^2}{4}} \qquad R_e' = \frac{R_e}{L}$$

$$= \frac{4\,(250\,A)^2\,(0.005\ \Omega/m)}{\pi\,(0.03\,m)^2} = 4.42\times 10^5\ W/m^3$$

$$T_s = 20 + \frac{(4.42\times10^5\ W/m^3)(0.015\,m)}{2\,(25\ W/m^2\,°C)} = 152.6\ °C$$

$$T_o = T_s + \frac{\dot{q}\, r_o^2}{4k} = 152.6 + \frac{(4.42\times10^5)(0.015)^2}{4\,(401)} = 152.7\ °C$$

Because of high k for copper, T is nearly

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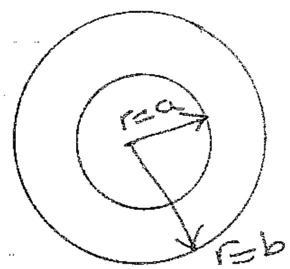
Example - 14

Develop an expression for the steady state temperature distribution $T(r)$ in a long, hallow cylinder, $a \le r \le b$, in which heat generation at a rate of

$$\dot{q}(r) = g_o(1 + Ar) \quad w/m^3$$

where g_o and A are constants, while the boundary surfaces at $r=a$ and $r=b$ are kept at zero temperature



$r=a \quad T=0$
$r=b \quad T=0$

Solution

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{K} = 0$$

$r=a \quad T=0$
$r=b \quad T=0$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{g_o(1+Ar)}{K} = 0$$

integrate twice

$$T(r) = -\frac{g_o}{4K}r^2 + C_1 \ln r + C_2$$

Using B.C. :

$$0 = -\frac{g_o}{4k}r_i^2 + C_1 \ln r_i + C_2$$

$$0 = -\frac{g_o}{4k} r_o^2 + C_1 \ln r_o + C_2$$

Solution gives

$$C_1 = \frac{g_o}{4k} \cdot \frac{r_o^2 - r_i^2}{\ln(r_o/r_i)}$$

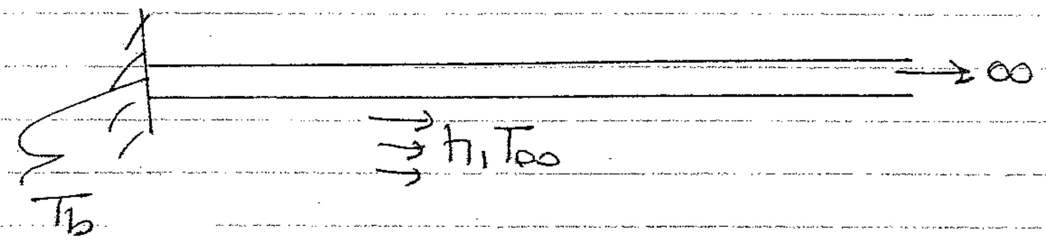
$$C_2 = \frac{g_o}{4k}\left[r_i^2 - \frac{r_o^2 - r_i^2}{\ln(r_o/r_i)} \ln(r_i)\right]$$

So

$$T(r) = \frac{g_o}{4k}\left[\frac{r_o^2 - r_i^2}{\ln(r_o/r_i)} \ln(r/r_i) - (r^2 - r_i^2)\right]$$

Example - 15

A very long fin (k=380 W/mK) has a diameter of 25 mm.



Fin extends from a surface at 120°C. The temperature of the surrounding air is 25°C and the heat transfer coefficient over the rod is 10 W/m²K. Calculate:

a) heat loss from the rod

b) how long the rod should be in order to be considered infinite?

solution

$D = 25\,mm = 0.025\,m$

$T_b = 120\,°C$

$h = 10\ W/m^2K$

$k = 380\ W/mK$

$T_\infty = 25°C$

a) $q = \sqrt{hPkA_c}\,(T_b - T_\infty)$

$P = \pi D = \pi(0.025\,m) = 0.07853\,m$

$A = \frac{\pi D^2}{4} = \left(\frac{\pi}{4}\right)(0.025\,m)^2 = 4.908 \times 10^{-4}\,m^2$

$Q = 36.36\ W$

b) for infinitely long fin $T_L = T_\infty$

$Q_{\substack{infinite\\fin}} = Q_{\substack{insulated\\fin}}$

$$\sqrt{hPkA_c}\,(T_b - T_\infty) = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(mL)$$

$$\tanh(mL) \geq 0.99$$

$$mL = 2.646$$

$$m = \sqrt{\frac{hP}{kA_c}} = 2.052$$

$$\therefore L = \frac{2.646}{2.052} = 1.29\ m$$

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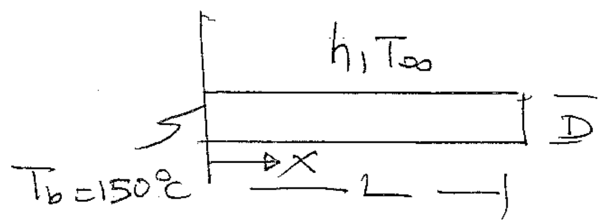
Example - 15

A steel rod of diameter D = 2c, length L = 25 cm, and thermal conductivity k = 50 W/m°C is exposed to ambient air at T_∞ = 20 °C with a heat transfer coefficient h = 64 W/m²°C. If one end of the rod is at temperature of 120°C, calculate heat loss from the ~~end~~ fin

Solution



L ≫ D so assume infinitely long fin model.

$$m^2 = \frac{hP}{kA} = \frac{4h}{kD} = 4\,(64) = 256$$

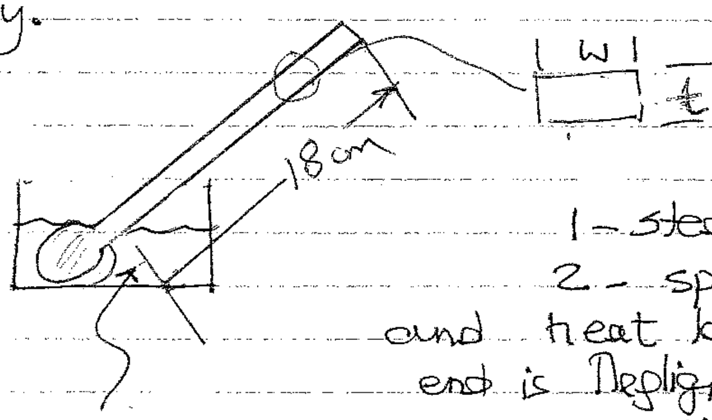
$$m = 16 \qquad mL = 16 \times 0.25 = 4$$

$$q = \theta_b \sqrt{PAkh} = \theta_b \sqrt{\pi D \left(\frac{\pi D^2}{4}\right) kh}$$

$$= 25.1 \quad W$$

Example-17

Consider a stainless steel spoon ($k = 15.1\ W/m\,K$) partially immersed in boiling water at 95°C in a kitchen at 25°C. The handle of the spoon has a cross section 0.2 cm x 1 cm and it extends 18 cm in the air from the free surface of water. If the local heat transfer on the exposed surface of spoon is $15\ W/m^2K$, calculate the temperature difference across exposed surface of the spoon handle. State your assumptions, if any.



$w = 1\,cm$
$t = 0.02\,cm$

Boiling water at 95°C

1- steady state
2- spoon is thin and heat loss from the end is Negligible
3- no radiation
4- constant properties
5- we assume that spoon partly in water and at 95°C. i.e $T_b = 95°C$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(mL)}$$

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$P = 2(w+t) = 2(1+0.2) = 2.4\,cm = 0.024\,m$$
$$A_c = wt = 0.2\,cm^2 = 0.2\times10^{-4}\,m^2$$

$m = 34.52\ m^{-1}$

$mL = 6.21$

$$T_L - T_\infty = \frac{95-25}{250} = 0.28\,°C$$

$T_L = 25.28\,°C$

$T_b - T_L = 95 - 25.28 = 69.72\,°C$