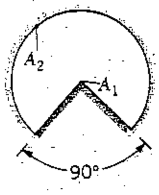
Chapter- 13

Examples

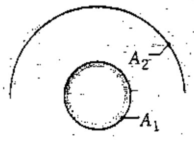
1) Determine F_{12} and F_{21} for the following configurations using the reciprocity theorem and other basic shape factor relations. Do not use tables or charts.

(a) Long duct



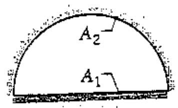
(a)

(b) Small sphere of area A_1 under a concentric hemisphere of area $A_2 = 2A_1$



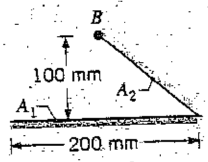
(b)

(c) Long duct. What is F_{22} for this case?



(c)

(d) Long inclined plates (point B is directly above the center of A_1)

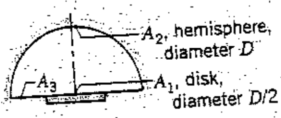


(d)

(e) Sphere lying on infinite plane

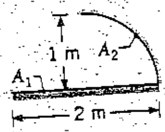
(e)

(f) Hemisphere–disk arrangement



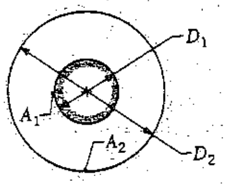
(f)

(g) Long, open channel

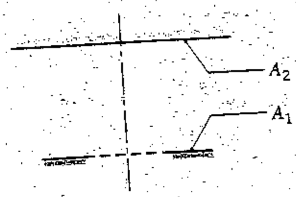


(g)

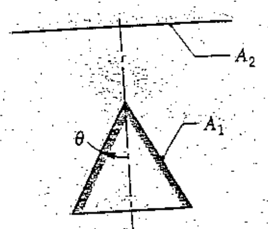
(h) Long concentric cylinders



(h)

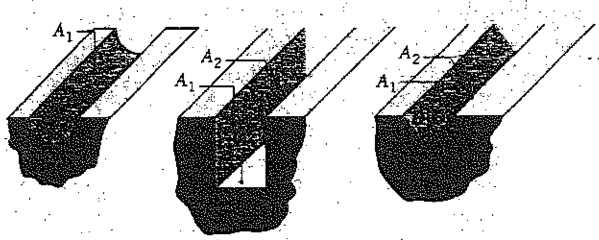


(a)



(b)

2) Consider the following grooves, each of width W, that have been machined from a solid block of material.

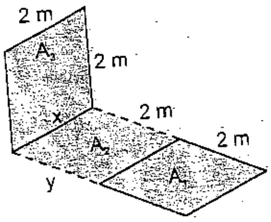


(a) For each case obtain an expression for the view factor of the groove with respect to the surroundings outside the groove.

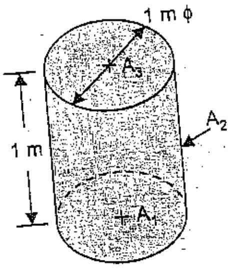
(b) For the V groove, obtain an expression for the view factor F_{12}, where A_1 and A_2 are opposite surfaces.

(c) If $H = 2W$ in the rectangular groove, what is the view factor F_{12}?

3) *Determine the shape factor from the surface 1 to surface 3 shown in Fig.* *(vertical plane and non touching horizontal surface).*



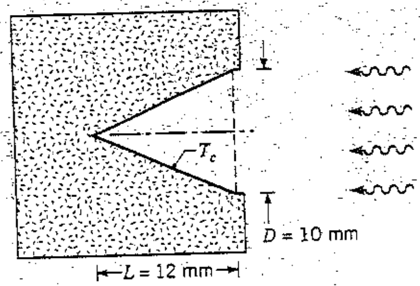
4) **Example** *Determine the shape factor from the base of a cylinder to the curved surface. Also find the shape factor from curved surface to base and the curved surface to itself.*



5) A circular ice rink 25 m in diameter is enclosed by a hemispherical dome 35 m in diameter. If ice and dome surfaces may be approximated as black bodies and are at 0° and 15 °C, respectively, what is the net rate of radiative transfer from the dome to the rink?

solution

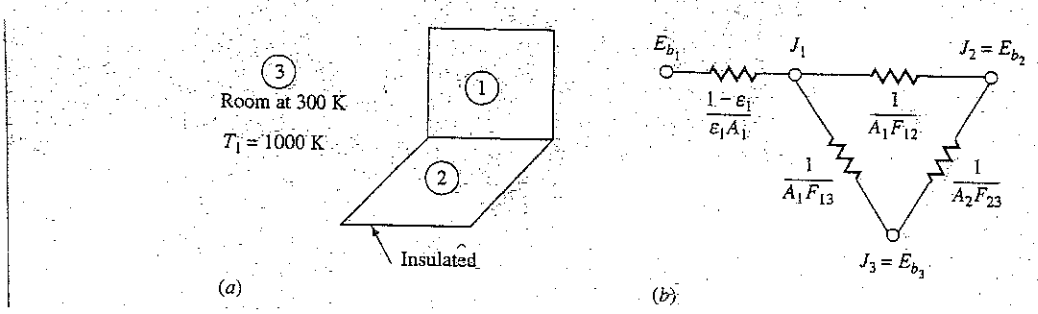
6) A meter to measure the optical power of a laser beam is constructed with a thin-walled, black conical cavity that is well insulated from its housing. The cavity has an opening of $D = 10$ mm and a depth of $L = 12$ mm. The meter housing and surroundings are at a temperature of 25.0°C.



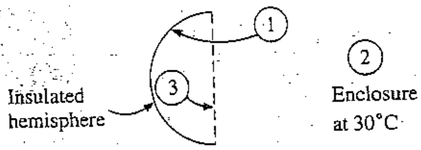
A fine-wire thermocouple attached to the surface indicates a temperature rise of 10.1°C when a laser beam is incident upon the meter. What is the optical (radiant) flux of the laser beam, G_o (W/m^2)?

7) **Example 1.** Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart, as shown in figure. One plate is maintained at 1000 °C and the other at 500 °C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27 °C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.

Example 8 Two rectangles 50 by 50 cm are placed perpendicularly with a common edge. One surface has T_1=1000 K, ε_1=0.6, while the other surface is insulated and in radiant balance with a large surrounding room at 300 K. Determine the temperature of the insulated surface and the heat lost by the surface at 1000 K.



Example 9 The 30 cm diameter hemisphere in figure is maintained at a constant temperature of 500 °C and insulated on its back side. The surface emissivity is 0.4. The opening exchanges radiant energy with a large enclosure at 30 °C. Calculate the net radiant exchange.



Example Two very large parallel planes with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction in heat transfer when a polished-aluminum radiation shield (ε=0.04) is placed between them.

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Example Two concentric cylinders having diameters of 10 and 20 cm have a length of 20 cm. They are at T_1=1000 K, ε_1=0.8, ε_2=0.2 and are located in a large room at 300 K. The outer cylinder is in radiant balance.

a) Calculate the shape factors
b) Calculate the temperature of the outer cylinder and the total heat lost by the inner cylinder.

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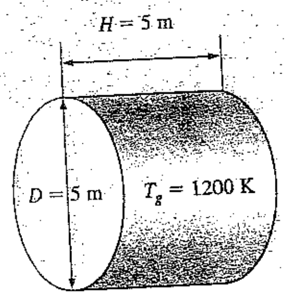
Example A cylindrical furnace whose height and diameter are 5 m. contains combustion gases at 1200 K and a total pressure of 2 atm. The composition of the combustion gases is determined by volumetric analysis to be 80 percent N_2, 8 percent H_2O, 7 percent O_2, and 5 percent CO_2. Determine the effective emissivity of the combustion gases.



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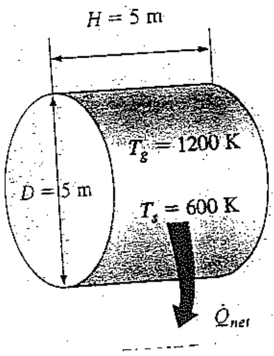
Example Reconsider the cylindrical furnace discussed in Example 6. For a wall temperature of 600 K, determine the absorptivity of the combustion gases and the rate of radiation heat transfer from the combustion gases to the furnace walls.



Chapter-13

Examples

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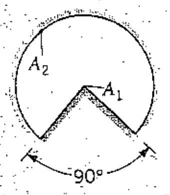
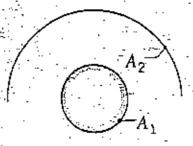


(a)

(b) Small sphere of area A_1 under a concentric hemisphere of area $A_2 = 2A_1$



(b)

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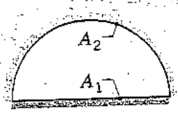
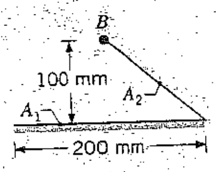


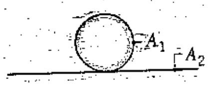
(c)

(d) Long inclined plates (point B is directly above the center of A_1)



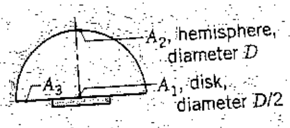
(d)

(e) Sphere lying on infinite plane



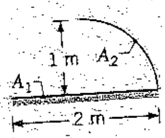
(e)

(f) Hemisphere–disk arrangement



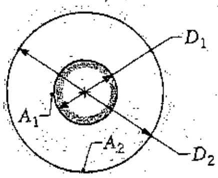
(f)

(g) Long, open channel

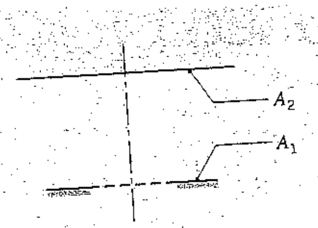


(g)

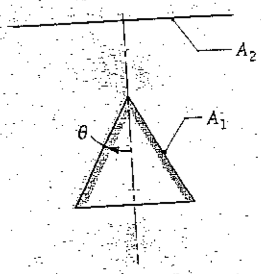
(h) Long concentric cylinders



(h)



(a)



(b)

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2)

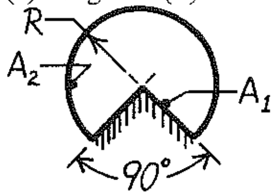
KNOWN: Various geometric shapes involving two areas A_1 and A_2.

FIND: Shape factors, F_{12} and F_{21}, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

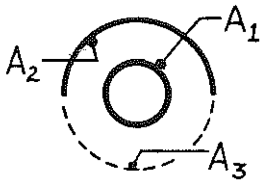
(a) Long duct (L):



By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$ <

(b) Small sphere, A_1, under concentric hemisphere, A_2, where $A_2 = 2A$

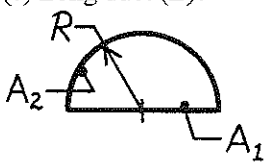


Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):

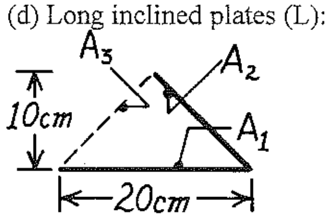


By inspection, $F_{12} = 1.0$

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

(d) Long inclined plates (L):

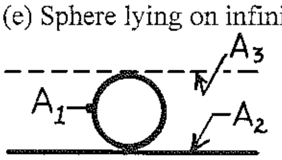


Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane



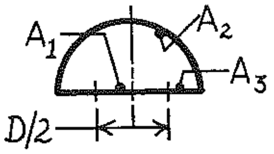
Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} \to 0$ since $A_2 \to \infty$. <

Continued .....

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.



By inspection, $F_{12} = 1.0$

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \text{ Hence, } F_{32} = 1.0.$$

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{ \left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] \Big/ \frac{\pi D^2}{2} \right\} 1.0 = 0.375.$$

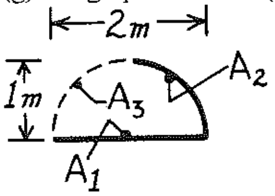
By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{ \frac{\pi}{4} \left[\frac{D}{2} \right]^2 \Big/ \frac{\pi D^2}{2} \right\} \times 1.0 = 0.125.$$

Summation rule for A_2, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):



Summation rule for A_1

$$F_{11} + F_{12} + F_{13} = 0$$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$.

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

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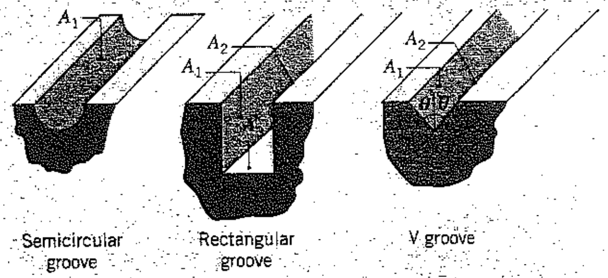
2)

Consider the following grooves, each of width W, that have been machined from a solid block of material.

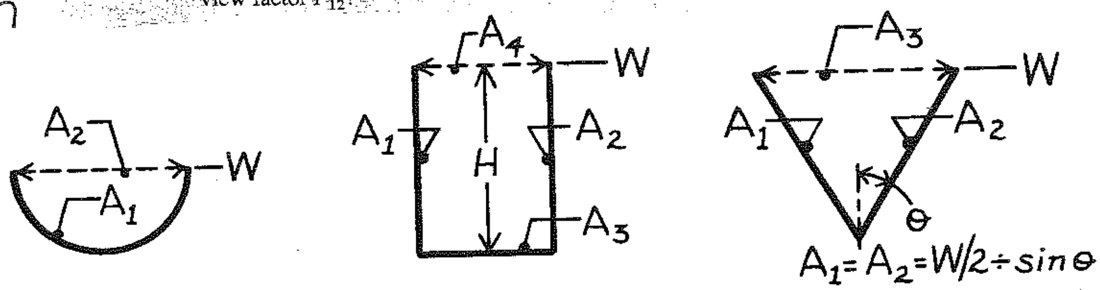


(a) For each case obtain an expression for the view factor of the groove with respect to the surroundings outside the groove.

(b) For the V groove, obtain an expression for the view factor F_{12}, where A_1 and A_2 are opposite surfaces.

(c) If $H = 2W$ in the rectangular groove, what is the view factor F_{12}?

solution



ASSUMPTIONS: (1) Diffuse surfaces, (2) Negligible end effects, "long grooves".

ANALYSIS: (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi.$$

Rectangular Groove:

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H).$$

V Groove:

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\frac{W/2}{\sin\theta} + \frac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $\quad F_{12} = 1 - F_{13} = 1 - \frac{A_3}{A_1} F_{31}.$

From Symmetry, $\quad F_{31} = 1/2.$

Hence, $F_{12} = 1 - \frac{W}{(W/2)/\sin\theta} \times \frac{1}{2}$ or $F_{12} = 1 - \sin\theta.$

(c) From Fig. 13.4, with X/L = H/W = 2 and Y/L → ∞,

$F_{12} \approx 0.62.$

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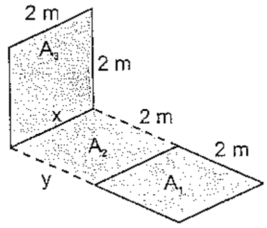
3) Determine the shape factor from the surface 1 to surface 3 shown in Fig. (vertical plane and non touching horizontal surface).



solution

$$(A_1+A_2)\,F_{(1+2)-3} = A_1 F_{13} + A_2 F_{2-3}$$

Using chart

$F_{(1+2)\,3} = ?$

$$\begin{cases} X=2 \\ Y=4 \\ L=2 \end{cases}$$

$$\left.\begin{aligned} \frac{X}{L} &= \frac{2}{2} = 1 \\ \frac{Y}{L} &= \frac{4}{2} = 2 \end{aligned}\right\} F_{(1+2)-3} = 0.11643$$

$F_{2-3} = ?$

$$\left.\begin{aligned} \frac{X}{L} &= \frac{2}{2} = 1 \\ \frac{Y}{L} &= \frac{2}{2} = 1 \end{aligned}\right\} F_{2-3} = 0.2$$

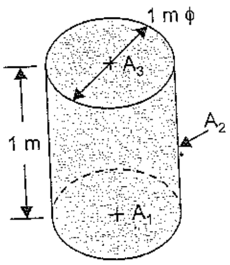
$$(2\times2+2\times2)(0.11643) = (2\times2)F_{1-3} + (2\times2)(0.2)$$

$$F_{1-3} = 0.032$$

Note that to find F_{3-1} use $A_1F_{1-3} = A_3 F_{3-1}$

So $F_{3-1} = 0.03288$

4) **Example** : *Determine the shape factor from the base of a cylinder to the curved surface. Also find the shape factor from curved surface to base and the curved surface to itself.*



$L = 1$

$r_j = 0.5$

$r_i = 0.5$

$\frac{L}{r_j} = \frac{1}{0.5} = 2$

$r_j/L = \frac{0.5}{1} = 0.5$

$F_{13} = 0.17$

Solution: The diameter is 1 m and height is also 1 m. The base (1) is enclosed by the top (3) and curved surface (2) (Fig. 13.22)

$$\therefore \qquad F_{1-2} + F_{1-3} = 1$$

F_{1-3} can be determined by using the chart for parallel disks. The ratio, diameter/distance between planes = 1

The corresponding value of shape factor **is 0.17.** Base to curved surface is

$$\therefore \qquad \mathbf{F_{1-2} = 1.0 - 0.17 = 0.83}$$

$A_1 = \pi r_1^2 = \pi/4$

$A_2 = 2\pi r_1 L$

Using reciprocity theorem

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$\frac{\pi \times 1^{\cancel{2}}}{4} \times \cancel{0.87}\ 0.83 = \pi \times \cancel{1 \times 1 \times} F_{2-1}$$

$(2\pi)(\frac{1}{2})(1) F_{21}$

$$\therefore \qquad \mathbf{F_{2-1} = \cancel{0.2175}}\ 0.2075$$

Considering the curved surface, as concave surface will intercept some radiation from the surface itself.

$$F_{2-1} + F_{2-3} + F_{2-2} = 1$$

As $\qquad \mathbf{F_{2-1}} = F_{2-3},\ F_{2-2} = 1 - 2 \times 0.2175 = \mathbf{0.565}$

Concave surfaces intercept part of radiation emitted by themselves. Here it intercepts more than half of the radiation.

$$\cancel{A_2} F_{2-1} = \cancel{A_2} F_{2-3} \Rightarrow F_{2-3} = 0.2075$$

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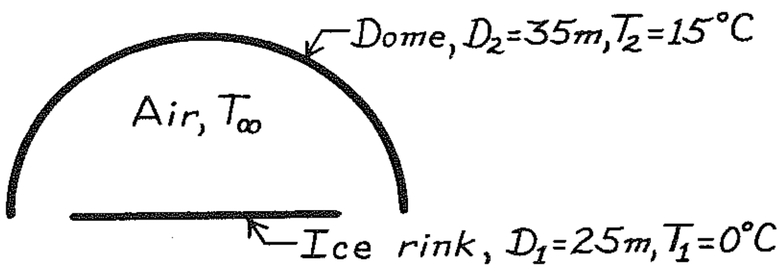
5) A circular ice rink 25 m in diameter is enclosed by a hemispherical dome 35 m in diameter. If ice and dome surfaces may be approximated as black bodies and are at 0° and 15°C, respectively, what is the net rate of radiative transfer from the dome to the rink?

solution

KNOWN: Temperature and diameters of a circular ice rink and a hemispherical dome.

FIND: Net rate of heat transfer to the ice due to radiation exchange with the dome.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for dome and ice.

ANALYSIS: From Eq. 13.14, $q_{ij} = A_i F_{ij}(J_i - J_j)$ where $J_i = \sigma T_i^4$ and $J_j = \sigma T_i^4$. Therefore,

$$q_{21} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right)$$

From reciprocity, $A_2 F_{21} = A_1 F_{12} = \left(\pi D_1^2 / 4\right) 1$

$$A_2 F_{21} = (\pi / 4)(25\ \text{m})^2 1 = 491\ \text{m}^2.$$

Hence

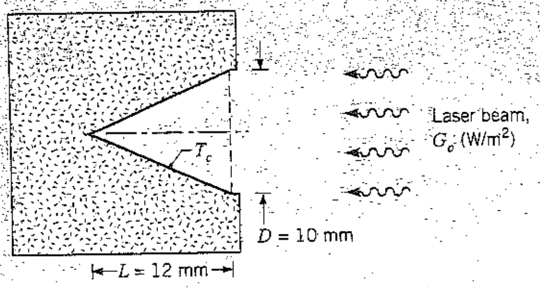
$$q_{21} = 491\ \text{m}^2 \left(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\right)\left[(288\ \text{K})^4 - (273\ \text{K})^4\right]$$

$$q_{21} = 3.69 \times 10^4\ \text{W}.$$ <

COMMENTS: If the air temperature, T_∞, exceeds T_1, there will also be heat transfer by convection to the ice. The radiation and convection transfer to the ice determine the heat load which must be handled by the cooling system.

6)

A meter to measure the optical power of a laser beam is constructed with a thin-walled, black conical cavity that is well insulated from its housing. The cavity has an opening of $D = 10$ mm and a depth of $L = 12$ mm. The meter housing and surroundings are at a temperature of 25.0°C.



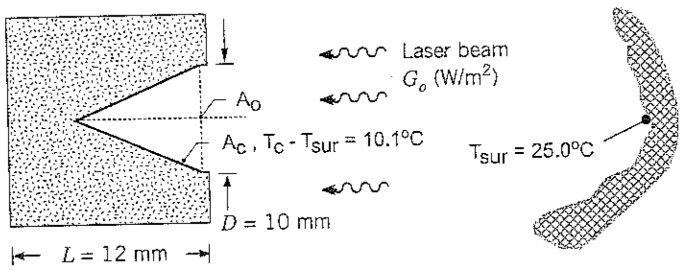
A fine-wire thermocouple attached to the surface indicates a temperature rise of 10.1°C when a laser beam is incident upon the meter. What is the optical (radiant) flux of the laser beam, G_o (W/m²)?

solution

KNOWN: Thin-walled, black conical cavity with opening D = 10 mm and depth of L = 12 mm that is well insulated from its surroundings. Temperature of meter housing and surroundings is 25.0°C.

FIND: Optical (radiant) flux of laser beam, G_o (W/m²), incident on the cavity when the fine-wire thermocouple indicates a temperature rise of 10.1°C.

SCHEMATIC:



ASSUMPTIONS: (1) Cavity surface is black and perfectly insulated from its mounting material in the meter, (2) Negligible convection heat transfer from the cavity surface, and (3) Surroundings are large, isothermal.

ANALYSIS: Perform an energy balance on the walls of the cavity considering absorption of the laser irradiation, absorption from the surroundings and emission.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$A_o\, G_o + A_o\, G_{sur} - A_o\, E_b\,(T_c) = 0$$

where $A_o = \pi D^2/4$ represents the opening of the cavity. All of the radiation entering or leaving the cavity passes through this hypothetical surface. Hence, we can treat the cavity as a black disk at T_c. Since $G_{sur} = E_b\,(T_{sur})$, and $E_b = \sigma T^4$ with $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴, the energy balance has the form

$$G_o + \sigma(25.0+273)^4 K^4 - \sigma(25.0+10.1+273)^4 K^4 = 0$$

$$G_o = 63.8\ \mathrm{W/m^2} \qquad <$$

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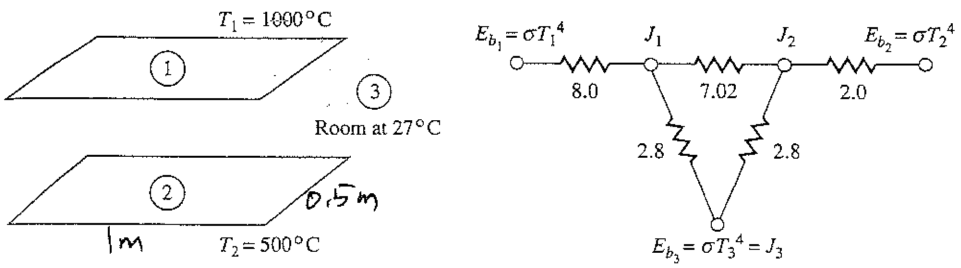
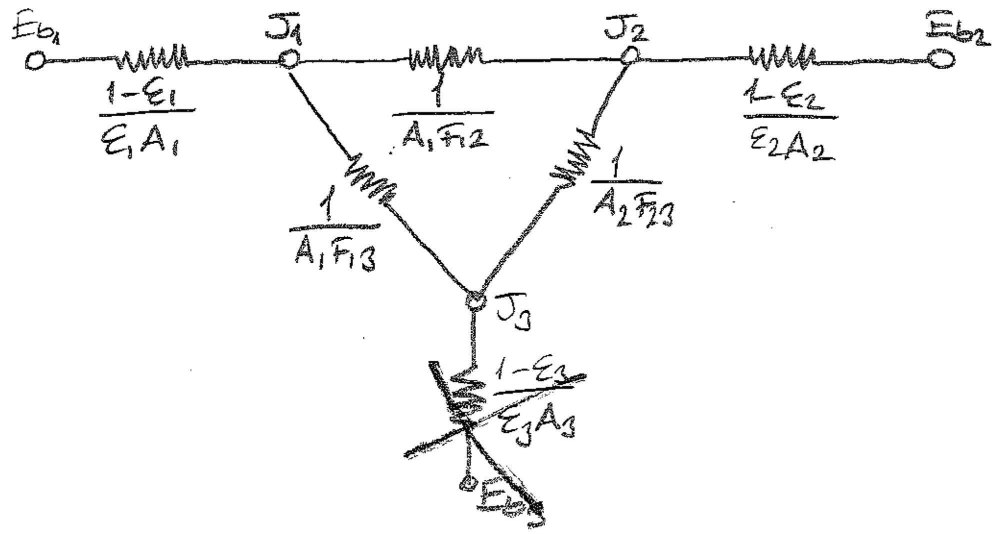
7) **Example 1.** Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart, as shown in figure. One plate is maintained at 1000 °C and the other at 500 °C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27 °C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.



(a) (b)

This is a three-body problem, the two plates and the room, so the radiation network is:



From the data of the problem:

$T_1 = 1000°C = 1273\,K$ $\qquad$ $A_1 = A_2 = 0.5\,m^2$

$T_2 = 500°C = 773\,K$ $\qquad$ $\varepsilon_1 = 0.2$

$T_3 = 27°C = 300\,K$ $\qquad$ $\varepsilon_2 = 0.5$

Because of the area of the room A_3 is very large, the resistance $(1-\varepsilon_3)/\varepsilon_3 A_3$ may be taken as zero and we obtain $E_{b3} = J_3$. The shape factor F_{12}:

$$\frac{Y}{L} = \frac{0.5}{0.5} = 1.0 \qquad \frac{X}{L} = \frac{1.0}{0.5} = 2.0$$

so that $F_{12} = 0.285$ (From Figure 13.4)

$F_{12} = 0.285 = F_{21}$

$F_{13} = 1 - F_{12} = 0.715$

$F_{23} = 1 - F_{21} = 0.715$

The resistances in the network are calculated as:

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.5)} = 8.0 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.5)} = 2.0$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.5)(0.285)} = 7.018 \qquad \frac{1}{A_1 F_{13}} = \frac{1}{(0.5)(0.715)} = 2.797$$

$$\frac{1}{A_2 F_{23}} = \frac{1}{(0.5)(0.715)} = 2.797$$

Taking the resistance $(1-\varepsilon_3)/\varepsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

node J_1: $$\frac{E_{b1} - J_1}{8.0} + \frac{J_2 - J_1}{7.018} + \frac{E_{b3} - J_1}{2.797} = 0 \qquad (a)$$

node J_2: $$\frac{J_1 - J_2}{7.018} + \frac{E_{b3} - J_2}{2.797} + \frac{E_{b2} - J_2}{2.0} = 0 \qquad (b)$$

$E_{b1} = \sigma T_1^4 = 148.87 \text{ kW/m}^2$

$E_{b2} = \sigma T_2^4 = 20.241 \text{ kW/m}^2$

$E_{b3} = \sigma T_3^4 = 0.4592 \text{ kW/m}^2$

Inserting the values of E_{b1}, E_{b2}, and E_{b3} into Equation (a) and (b), we have two equations and two unknowns J_1 and J_2 that may be solved simultaneously to give

$J_1 = 33.469 \text{ kW/m}^2$ $\quad$ $J_2 = 15.054 \text{ kW/m}^2$

The total heat lost by plate 1 is

$$q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{148.87 - 33.469}{8.0} = 14.425 \text{ kW}$$

and the total heat lost by plate 2 is

$$q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \frac{20.241 - 15.054}{2.0} = 2.594 \text{ kW}$$

The total heat received by the room is

$$q_3 = \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_2 - J_3}{1/A_2 F_{23}}$$

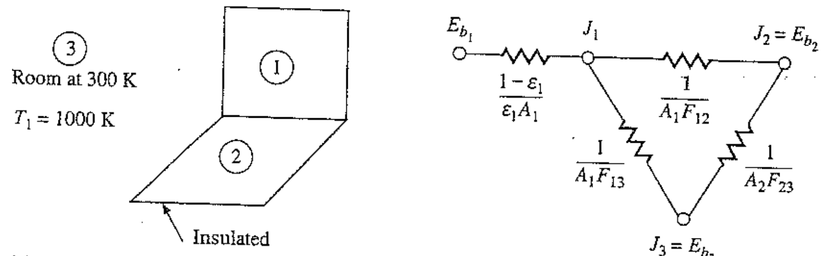
$$= \frac{33.469 - 0.4592}{2.797} + \frac{15.054 - 0.4592}{2.797} = 17.020 \text{ kW}$$

From an overall-balance standpoint we must have

$$q_3 = q_1 + q_2$$

because the net energy lost by both plates must be absorbed by the room.

Example 8 Two rectangles 50 by 50 cm are placed perpendicularly with a common edge. One surface has T_1=1000 K, ε_1=0.6, while the other surface is insulated and in radiant balance with a large surrounding room at 300 K. Determine the temperature of the insulated surface and the heat lost by the surface at 1000 K.



(a) (b)

The radiation network is shown in figure where surface 3 is the room and surface 2 is the insulated surface. Note that $J_3 = E_{b_3}$ because the room is large and $(1-\varepsilon_3)/\varepsilon_3 A_3$ aproaches zero. Because surface 2 is insulated it has zero heat transfer and $J_2 = E_{b_2}$. J_2 "floats" in the network and is determined from the overall radiant balance.

From Figure 13.6 the shape factors are

$$F_{12} = 0.2 = F_{21}$$

Because $F_{11} = 0$ and $F_{22} = 0$ we have

$$F_{12} + F_{13} = 1.0 \quad \text{and} \quad F_{13} = 1 - 0.2 = 0.8 = F_{23}$$

$$A_1 = A_2 = (0.5)^2 = 0.25 m^2$$

The resistances are

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{0.4}{(0.6)(0.25)} = 2.667$$

$$\frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = \frac{1}{(0.25)(0.8)} = 5.0$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.25)(0.2)} = 20.0$$

We also have

$$E_{b_1} = (5.669\times10^{-8})(1000)^4 = 5.669\times10^4 \text{ W/m}^2$$

$$J_3 = E_{b_3} = (5.669\times10^{-8})(300)^4 = 459.2 \text{ W/m}^2$$

The overall circuit is a series-parallel arrangement and the heat transfer is

$$q = \frac{E_{b_1} - E_{b_3}}{R_{equiv}}$$

We have

$$R_{equiv} = 2.667 + \frac{1}{\frac{1}{5} + 1/(20+5)} = 6.833$$

and

$$q = \frac{56690 - 459.2}{6.833} = 8.229 \text{ kW}$$

This heat transfer can also be written

$$q = \frac{E_{b_1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1}$$

Inserting the values we obtain

$$J_1 = 34745 \text{ W/m}^2$$

The value of J_2 is determined from proportioning the resistances between J_1 and J_3, so that

$$\frac{J_1 - J_2}{20} = \frac{J_1 - J_3}{20+5}$$

and

$$J_2 = 7316 = E_{b_2} = \sigma T_2^4$$

Finally, we obtain the temperature of the insulated surface as

$$T_2 = \left(\frac{7316}{5.669\times10^{-8}}\right)^{1/4} = 599.4 \text{ K}$$

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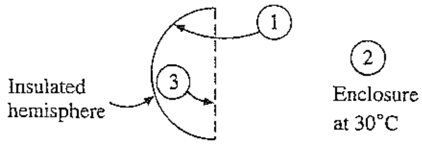
Example 9: The 30 cm diameter hemisphere in figure is maintained at a constant temperature of 500 °C and insulated on its back side. The surface emissivity is 0.4. The opening exchanges radiant energy with a large enclosure at 30 °C. Calculate the net radiant exchange.



This is an object completely surrounded by a large enclosure but the inside surface of the sphere is not convex. In the figure we take the inside of the sphere as surface 1 and the enclosure as surface 2. We also create an imaginary surface 3 covering the opening. We actually have a two-surface problem (surfaces 1 and 2).

$$E_{b1} = \sigma T_1^4 = \sigma (773)^4 = 20241 \text{ W/m}^2$$

$$E_{b2} = \sigma T_2^4 = \sigma (303)^4 = 478 \text{ W/m}^2$$

$$A_1 = 2\pi r^2 = (2)\pi(0.15)^2 = 0.1414 \text{ m}^2$$

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{0.6}{(0.4)(0.1414)} = 10.61$$

$$A_2 \to \infty$$

so that

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} \to 0$$

Now, at this point we recognize that all of the radiation leaving surface 1 which will eventually arrive at enclosure 2 will also hit the imaginary surface 3 (ie., $F_{12} = F_{13}$). We also recognize that

$$A_1 F_{13} = A_3 F_{31}$$

But, $F_{31} = 1.0$ so that

$$F_{13} = F_{12} = \frac{A_3}{A_1} = \frac{\pi r^2}{2\pi r^2} = 0.5$$

Then $1/A_1F_{12} = 1/(0.1414)(0.5) = 14.14$ and we can calculate the heat transfer:

$$q = \frac{\sigma(T_1^4 - T_2^4)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

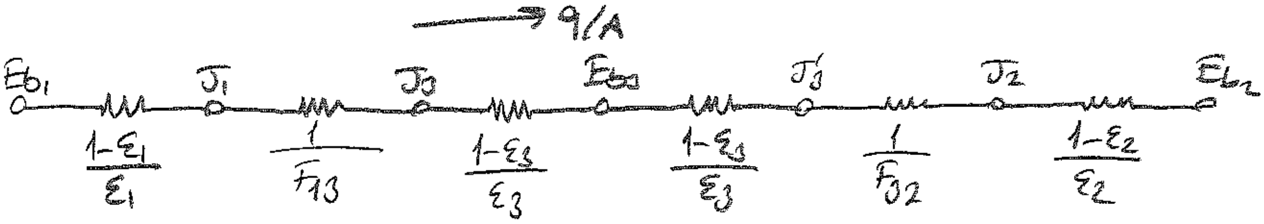
$$q = \frac{20241 - 478}{10.61 + 14.14 + 0} = 799\,W$$

Example 4. Two very large parallel planes with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction in heat transfer when a polished-aluminum radiation shield (ε=0.04) is placed between them.

The heat transfer without the shield is given by

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = 0.279\sigma(T_1^4 - T_2^4)$$

The radiation network for the problem



$$\frac{1-\varepsilon_1}{\varepsilon_1} = \frac{1-0.3}{0.3} = 2.333$$

$$\frac{1-\varepsilon_3}{\varepsilon_3} = \frac{1-0.04}{0.04} = 24.0$$

$$\frac{1-\varepsilon_2}{\varepsilon_2} = \frac{1-0.8}{0.8} = 0.25$$

The total resistance with the shield is

$$2.333 + (2)(24.0) + (2)(1) + 0.25 = 52.583$$

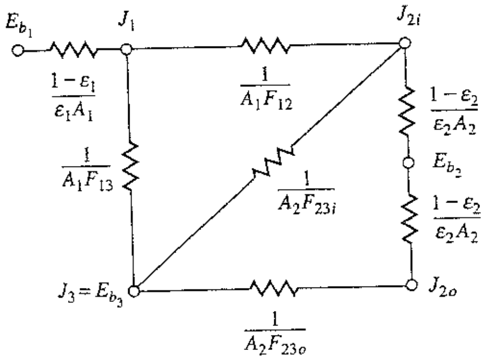
and the heat transfer is

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{52.583} = 0.01902\sigma(T_1^4 - T_2^4)$$

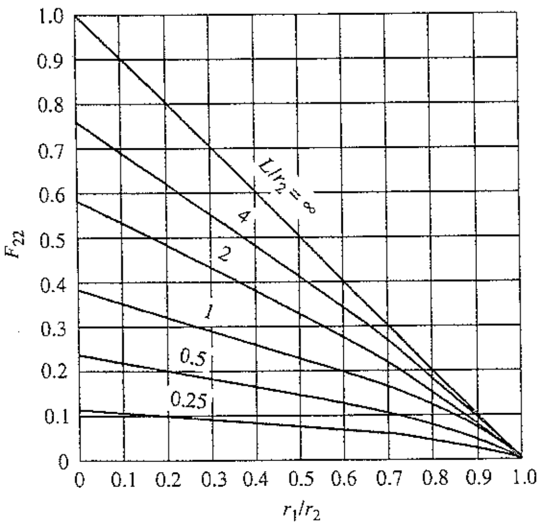
So that the heat transfer is reduced by 93.2 percent.

Example 1.1 Two concentric cylinders having diameters of 10 and 20 cm have a length of 20 cm. They are at T_1=1000 K, ε_1=0.8, ε_2=0.2 and are located in a large room at 300 K. The outer cylinder is in radiant balance.

a) Calculate the shape factors
b) Calculate the temperature of the outer cylinder and the total heat lost by the inner cylinder.



a) We use the nomenclature of figures below and designate the open ends os surfaces 3 and 4.



(a)

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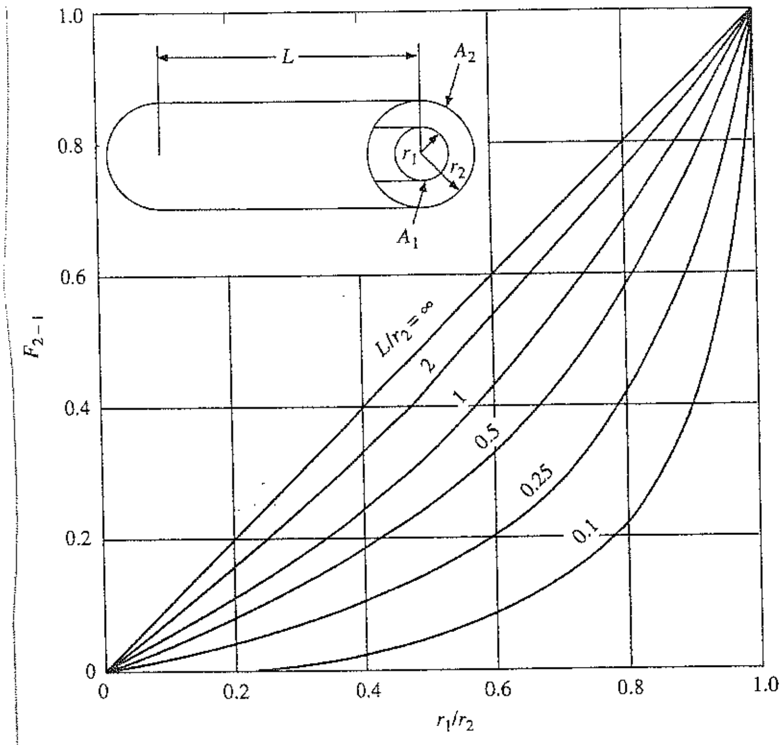

(b)

We have $L/r_2 = 20/10 = 2.0$ and $r_1/r_2 = 0.5$; so from this figure we obtain

$$F_{21} = 0.4126 \qquad F_{22} = 0.3286$$

Using the reciprocity relation, we have

$$A_1 F_{12} = A_2 F_{21} \quad \text{and} \quad F_{12} = (d_2/d_1)F_{21} = (20/10)(0.4126) = 0.8253$$

For surface 2 we have

$$F_{21} + F_{22} + F_{23} + F_{24} = 1.0$$

From symmetry $F_{23} = F_{24} = \frac{1}{2}(1 - 0.4126 - 0.3286) = 0.1294$

Using reciprocity again,

$$A_2 F_{23} = A_3 F_{32}$$

and

$$F_{32} = \frac{\pi(20)(20)}{\pi(20^2 - 10^2)/4}\, 0.1294 = 0.6901$$

We observe that $F_{11} = F_{33} = F_{44} = 0$ and for surface 3

$$F_{31} + F_{32} + F_{34} = 1.0$$

So, if F_{31} can be determined, we can calculate the desired quantity F_{34}. For surface 1

$F_{12} + F_{13} + F_{14} = 1.0$

and from symmetry $F_{13} = F_{14}$ so that

$F_{13} = \left(\frac{1}{2}\right)(1 - 0.8253) = 0.0874$

Using reciprocity gives

$$A_1 F_{13} = A_3 F_{31}$$

$$F_{31} = \frac{\pi(10)(20)}{\pi(20^2 - 10^2)/4} 0.0874 = 0.233$$

Then,

$$F_{31} + F_{32} + F_{34} = 1.0$$

$$F_{34} = 1 - 0.233 - 0.6901 = 0.0769$$

b) The room is designated as surface 3 and $J_3 = E_{b3}$, because the room is very large (i.e, its surface resistance is very small). In this problem we must consider the inside and outside of surface 2 and thus have subscripts i and o to designate the respective quantities. The shape factors can be obtained from (a).

$$F_{12} = 0.8253 \qquad F_{13} = 0.1747$$

$$F_{23i} = (2)(0.1294) = 0.2588 \quad F_{23o} = 1.0$$

Also,

$$A_1 = \pi(0.1)(0.2) = 0.06283 \, m^2$$

$$A_2 = \pi(0.2)(0.2) = 0.12566 \, m^2$$

$$E_{b1} = (5.669 \times 10^{-8})(1000)^4 = 5.669 \times 10^4 \ W/m^2$$

$$E_{b3} = (5.669 \times 10^{-8})(300)^4 = 459.2 \ W/m^2$$

and the resistances may be calculated as

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = 3.979 \qquad \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = 31.83$$

$$\frac{1}{A_1F_{12}} = 19.28 \qquad \frac{1}{A_2F_{23i}} = 30.75$$

$$\frac{1}{A_2F_{23o}} = 7.958 \qquad \frac{1}{A_1F_{13}} = 91.1$$

The network could be solved as a series-parallel circuit to obtain the heat transfer, but we will need the radiosities anyway, so we set up three nodal equations to solve for J_1, J_{2i}, and J_{2o}. We sum the currents into each node and set them equal to zero:

node J_1 :
$$\frac{E_{b1}-J_1}{3.979} + \frac{E_{b3}-J_1}{91.1} + \frac{J_{2i}-J_1}{19.28} = 0$$

node J_{2i} :
$$\frac{J_1-J_{2i}}{19.28} + \frac{E_{b3}-J_{2i}}{30.75} + \frac{J_{2o}-J_{2i}}{(2)(31.83)} = 0$$

node J_{2o} :
$$\frac{E_{b3}-J_{2o}}{7.958} + \frac{J_{2i}-J_{2o}}{(2)(31.83)} = 0$$

These equations have the solution

$J_1 = 49732\ W/m^2$

$J_{2i} = 26444\ W/m^2$

$J_{2o} = 3346\ W/m^2$

The heat transfer is then calculated from

$$q = \frac{E_{b1}-J_1}{(1-\varepsilon_1)/\varepsilon_1A_1} = \frac{56690-49732}{3.979} = 1749\ W$$

From the network we see that

$$E_{b2} = \frac{J_{2i}+J_{2o}}{2} = \frac{26444+3346}{2} = 14895\ W/m^2$$

and

$$T_2 = \left(\frac{14895}{5.669\times10^{-8}}\right)^{1/4} = 716\ K$$

If the outer cylinder had not been in place acting as a "shield" the heat lost from cylinder 1 could have been

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calculated from:

$$q = \varepsilon_1 A_1 (E_{b1} - E_{b3})$$

$$= (0.8)(0.06283)(56690 - 459.2) = 2826\ W$$

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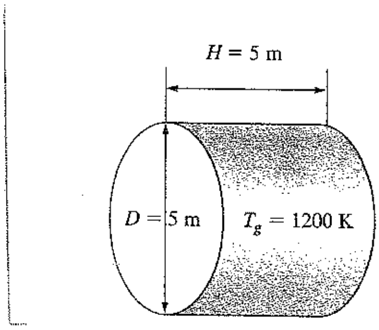
Example A cylindrical furnace whose height and diameter are 5 m contains combustion gases at 1200 K and a total pressure of 2 atm. The composition of the combustion gases is determined by volumetric analysis to be 80 percent N_2, 8 percent H_2O, 7 percent O_2, and 5 percent CO_2. Determine the effective emissivity of the combustion gases.



The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.05 (2 atm) = 0.10 atm$$

$$P_w = y_{H_2O} P = 0.08 (2 atm) = 0.16 atm$$

The mean beam length for a cylinder of equal diameter and height for radiation emitted to all surfaces is, from Table 13.4,

$$L_e = 0.60 D = (0.60)(5 m) = 3 m,$$

Then,

$$P_c L_e = (0.10 atm)(3 m) = 0.30 m.atm$$

$$P_w L_e = (0.16 atm)(3 m) = 0.48 m.atm$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of $T_g = 1200 K$ and 1 atm are, from Figures 13.16 and 13.18,

$$\varepsilon_{c, 1atm} = 0.16 \quad \text{and} \quad \varepsilon_{w, 1atm} = 0.23$$

These are the base emissivity values at 1 atm, and they need to be corrected for the 2 atm total pressure. Noting that

$(P_w+P)/2 = (0.16+2)/2 = 1.08$ atm, the pressure correction factors are, from Figure 13.17 and 13.19,

$$C_c = 1.1 \quad \text{and} \quad C_w = 1.4$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g = 1200K$ is from Figure 13.20,

$$\left.\begin{aligned} P_cL_e + P_wL_e &= 0.98 + 1.57 = 2.55 \\ \frac{P_w}{P_w+P_c} &= \frac{0.16}{0.16+0.10} = 0.615 \end{aligned}\right\} \Delta\varepsilon = 0.048$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c\,\varepsilon_{c,1atm} + C_w\,\varepsilon_{w,1atm} - \Delta\varepsilon = 1.1\times0.16 + 1.4\times0.23 - 0.048$$
$$= 0.45$$

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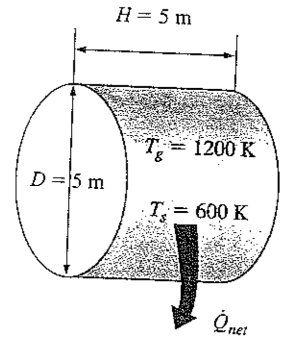
Example . Reconsider the cylindrical furnace discussed in Example 6. For a wall temperature of 600 K, determine the absorptivity of the combustion gases and the rate of radiation heat transfer from the combustion gases to the furnace walls.



The average emissivity of the combustion gases at the gas temperature of $T_g = 1200$ K was determined in the preceding example to be $\varepsilon_g = 0.45$
For a source temperature of $T_s = 600$K, the absorbtivity of the gas is again determined using the emissivity charts as

$$P_c L_e \frac{T_s}{T_g} = (0.10\,atm)(3m)\frac{600K}{1200K} = 0.15\,m.atm$$

$$P_w L_e \frac{T_s}{T_g} = (0.16\,atm)(3m)\frac{600K}{1200K} = 0.24\,m.atm$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of $T_s = 600$K and 1atm are, from Figure 13.16 and 13.18,

$$\varepsilon_{c,1atm} = 0.11 \quad \text{and} \quad \varepsilon_{w,1atm} = 0.25$$

The pressure correction factors were determined in the preceding example to be $C_c = 1.1$ and $C_w = 1.4$, and they do not change with surface temperature. Then the absorbtivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1atm} = (1.1)\left(\frac{1200K}{600K}\right)^{0.65}(0.11) = 0.19$$

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45}\varepsilon_{w,1atm} = (1.4)\left(\frac{1200K}{600K}\right)^{0.45}(0.25) = 0.48$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Figure 13.20 at $T = T_s = 600K$ instead of $T_g = 1200K$. There is no chart for 600K in the figure, but we can read $\Delta\varepsilon$ values at 400K and 800K, and take their average. At $P_w/(P_w+P_c) = 0.615$ and $P_cL_e + P_wL_e = 2.55$ we read $\Delta\varepsilon = 0.027$. Then the absorbtivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.19 + 0.48 - 0.027 = 0.64$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(5m)(5m) + 2\frac{\pi(5m)^2}{4} = 118 m^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{net} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (118\,m^2)(5.67\times10^{-8}\,W/m^2\cdot K^4)[0.45(1200K)^4 - 0.64(600K)^4]\\ &= 2.79\times10^4\,W\end{aligned}$$