Chapter 12

WORKED EXAMPLES

1) Calculate the blackbody emissive power E_b and the peak wavelength λ_{max} for surfaces at 300 K, 1000 K, 2000 K, and 5000 K.

$E_b = \sigma T^4$, $\sigma = 5.6697 \times 10^{-8}\ W/m^2K^4$

$E_b = \sigma (300)^4 = 459.2\ W/m^2$ for 300 K

$E_b = \sigma (1000)^4 = 5.67 \times 10^4\ W/m^2$ for 1000 K

$E_b = \sigma (2000)^4 = 9.07 \times 10^4\ W/m^2$ for 2000 K

$E_b = \sigma (5000)^4 = 3.54 \times 10^7\ W/m^2$ for 5000 K

$\lambda_{max} T = 2897.8\ \mu m K$

At 300 K $\lambda_{max} = 9.66\ \mu m$

At 1000 K $\lambda_{max} = 2.90\ \mu m$

At 2000 K $\lambda_{max} = 1.45\ \mu m$

At 5000 K $\lambda_{max} = 0.58\ \mu m$

2) A long 5 cm diameter cylinder has a surface temperature of 900 K. Assuming that the cylinder radiates as a blackbody, find the total blackbody emissive power and the radiation emitted per meter of cylinder length.

$E_b = \sigma T^4 = 37.2\ kW/m^2$

Blackbody emissive power, E_b, is the radiative heat flux;

$q'' = E_b = \frac{\dot{Q}}{A}$

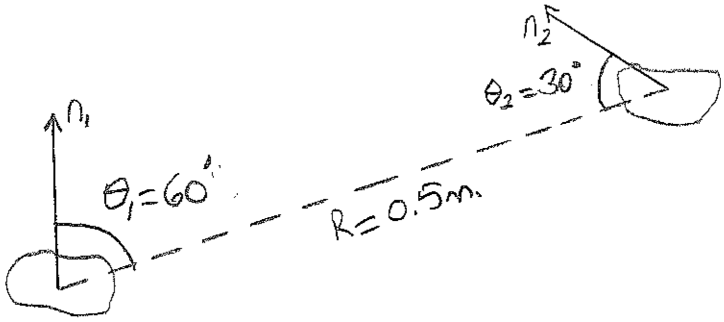
$\frac{\dot{Q}}{L} = \pi D E_b = \pi D \sigma T^4 = 5842\ W/m$

3) Consider a small surface $A_1=10^{-4}$ m^2 which emits diffusely with a total hemispherical emissive power of $E_1=5\times10^4$ W/m^2.

a) At what rate is this emission intercepted by a small surface of area $A_2=5\times10^4$ m^2 which is oriented as shown.

b) What is the irradiation G_2 on A_2?

10^{-4}



Assumptions:

1) Surface A_1 emits diffusely

2) A_1 is approximated as a differential surface area and $A_2/R \ll 1$

a) Emission from A_1 is intercepted by A_2

$q_{1-2}=I_{e1}(\theta,\phi)A_1\cos\theta_1\, d\omega_{2-1}$

A_1 is diffuse, $I_{e1}=\frac{E_1}{\pi}$

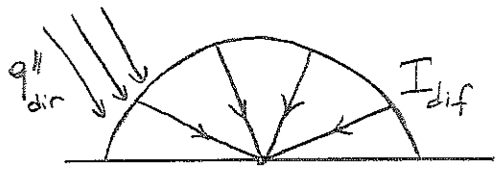
solid angle subtended by A_2 with respect to A_1 is

$d\omega_{21}=A_2\cos\theta_2/R^2$

$q_{1-2}=\frac{E_1}{\pi}A_1\cos\theta_1\cdot\frac{A_2\cos\theta_2}{R^2}=1.378\times10^{-3}$ W

b) $G_2=\frac{q_{12}}{A_2}=2.76$ W/m^2

4) Consider clear sky conditions for which the direct radiation is incident at θ=30° with a total flux (based on area normal to sun rays) of q''_{dir}=1000 W/m^2 and total intensity of diffuse radiation is I_{dif}=70 W/m^2.sr. What is the total solar irradiation at the earth's surface?



Irradiation G is based on actual area

$$G_{dir} = q''_{dir} \cos\theta$$

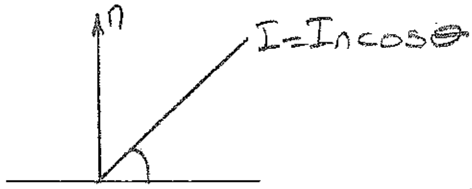
and

$$G_{dif} = \pi I_{dif}$$

$$G = G_{dir} + G_{dif} = q''_{dir} \cos\theta + \pi I_{dif}$$

$$= (1000)(0.866) + (\pi)(70) = 1086 \text{ W/m}^2$$

5) Directional distribution of the solar radiation on a surface is given by; $I_i=I_n \cos\theta$ where $I_n=80$ W/m^2.sr is total intensity of radiation in a direction normal to surface and θ is zenith angle. What is the solar irradiation on the surface?

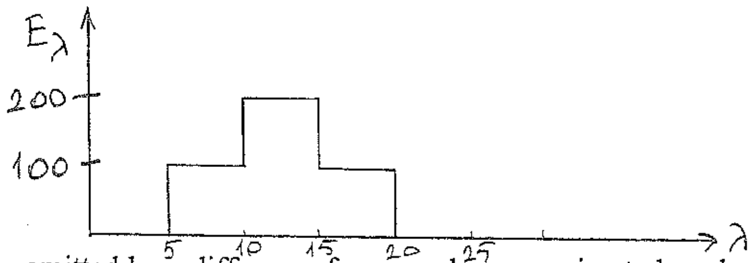


$$G = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta)\cos\theta\sin\theta\, d\theta\, d\phi$$

$$= 2\pi I_n \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta$$

$$= 167.6 \text{ W/m}^2$$

6) The spectral distribution of the radiation



emitted by a diffuse surface may be approximated as show in above.

a) What is the total emissive power?

b) What is the total intensity of the radiation emitted in the normal direction and at an angle of 30° from the normal?

c) Determine the fraction of emissive power leaving the surface in the direction: $\frac{\Pi}{4} \le \theta \le \frac{\Pi}{2}$.

Assume : diffuse emission

$$a)\ E = \int_0^\infty E_\lambda(\lambda)d\lambda = \int_0^5 0\,d\lambda + \int_5^{10} 100\,d\lambda + \int_{10}^{15} 200\,d\lambda + \int_{15}^{20} 100\,d\lambda + \int_{20}^{\infty} 0\,d\lambda$$

$$= 2000\ W/m^2$$

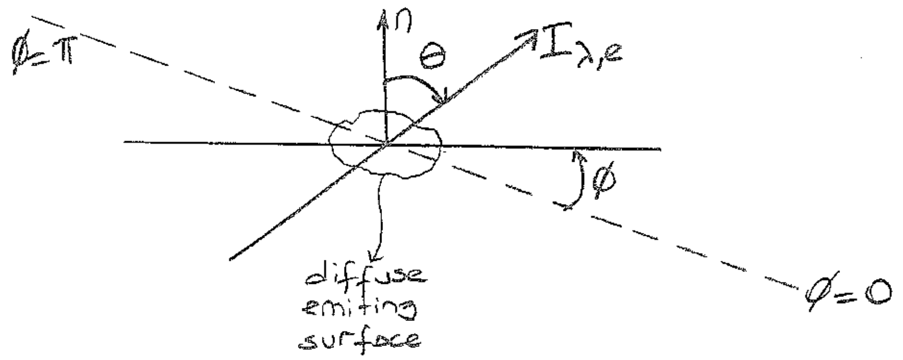
$$b)\ I_e = \frac{E}{\pi} = \frac{2000}{\pi} = 637\ W/m^2.sr$$

$$c)\ \frac{E(\pi/4 \to \pi/2)}{E} = \frac{\int_0^{2\pi}\int_{\pi/4}^{\pi/2} I_e \cos\theta \sin\theta\, d\theta}{\pi I_e}$$

$$= \frac{\int_{\pi/4}^{\pi/2} \cos\theta\sin\theta\, d\theta \int_0^{2\pi} d\phi}{\pi} = \frac{1}{\pi}\left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} \phi\Big|_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{1}{2}(1^2 - 0.707^2)(2\pi - 0)\right] = 0.276$$

7) Determine the fraction at the total, hemispherical emissive power that leaves a diffuse surface in the directions $\frac{\Pi}{4} \leq \theta \leq \frac{\Pi}{2}$ and $0 \leq \phi \leq \Pi$.



Assume: diffuse emitting surface

$$E = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi\, d\lambda$$

for diffuse surface $I_{\lambda,e}(\lambda,\theta,\phi)$ is independent of direction

$E = \pi I_e$

$$\Delta E = \int_0^{\infty} I_{\lambda,e}(\lambda)\, d\lambda \left[\int_0^{\pi} d\phi\right]\left[\int_{\pi/4}^{\pi/2} \cos\theta\sin\theta\, d\theta\right]$$

$$= I_e \left[\phi\right]_0^{\pi} \left[\frac{\sin^2\theta}{2}\right]_{\pi/4}^{\pi/2} = I_e(\pi)\left[\frac{1}{2}(1-0.707^2)\right]$$

$$= \frac{\pi}{4} I_e$$

$$\frac{\Delta E}{E} = \frac{0.25\pi I_e}{\pi I_e} = 0.25$$

8) A spherical aluminum shell of inside diameter D=2 m is evacuated and is used as a radiation test chamber. If the inner surface is coated with a carbon black and maintained at 600 K, what is the irradiation on a small test surface placed in the chamber?

Assume: 1) isothermal walls, 2) test surface area is small compared to enclosure surface.

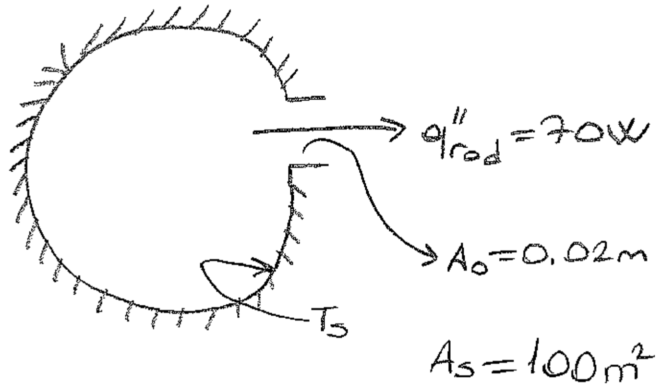
$G_1 = E_b(T_s)$

Because isothermal sphere is an enclosure behaving as a blackbody

$G_1 = \sigma T_s^4 = 7348\ W/m^2.$

9) An enclosure has an inside area of 100 m^2, and its surface is black and is maintained at a constant temperature. A small opening in the enclosure has an area of 0.02 m^2. The radiant power emitted from this opening is 70 W. What is the temperature of the interior enclosure wall? If the interior surface is maintained at this temperature, but is now polished, what will be the value of the radiant power emitted from the opening?

Assume that enclosure is isothermal and $A_o < A_s$

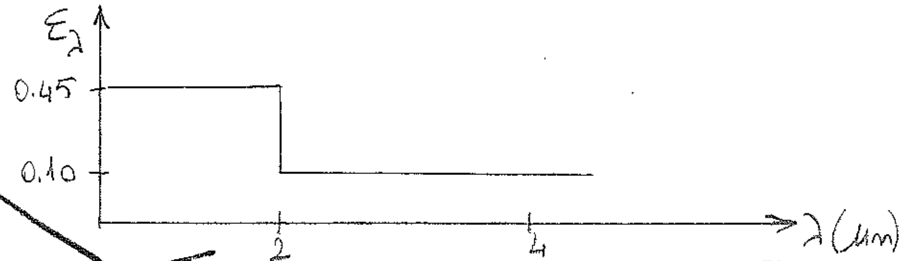


$$q_{rad} = A_o E_b(T_s) = A_o \sigma T_s^4$$

$$T_s = \sqrt[4]{\frac{q_{rad}}{\sigma A_o}} = \left[\frac{70}{(0.02)(5.67\times10^{-8})}\right]^{1/4} = 498\,K$$

Radiation depends on temperature only.

10) The spectral, hemispherical emissivity of tungsten may be approximated by the distribution depicted below. consider a cylindrical tungusten filament that is of diameter D=0.8 mm and length L=20 mm. The filament is enclosed in an evacuated bulb and is heated by electrical current to steady state temperature of 2900K.



a) What is the total hemispherical emissivity when the filament temperature is 2900 K

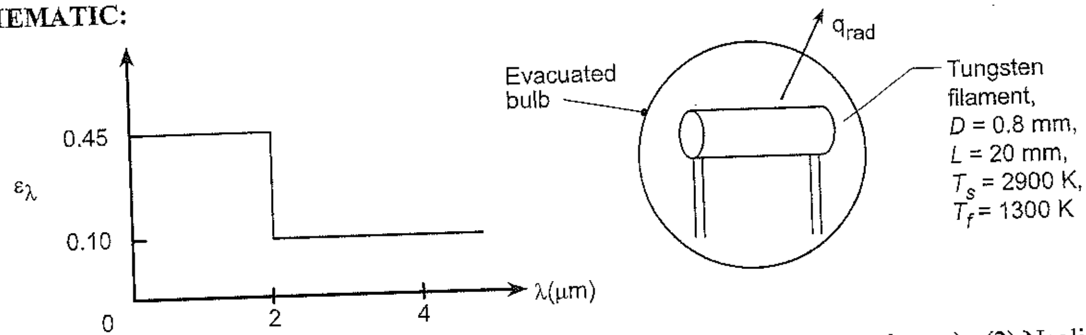
b) Assuming the surroundings are at 300 K, what is the initial rate of cooling of the filament when the current is switched off?

solution

KNOWN: Spectral emissivity, dimensions and initial temperature of a tungsten filament.

FIND: (a) Total hemispherical emissivity, ε, when filament temperature is $T_s = 2900$ K; (b) Initial rate of cooling, dT_s/dt, assuming the surroundings are at $T_{sur} = 300$ K when the current is switched off; (c) Compute and plot ε as a function of T_s for the range $1300 \le T_s \le 2900$ K; and (d) Time required for the filament to cool from 2900 to 1300 K.

SCHEMATIC:



ASSUMPTIONS: (1) Filament temperature is uniform at any time (lumped capacitance), (2) Negligible heat loss by conduction through the support posts, (3) Surroundings large compared to the filament, (4) Spectral emissivity, density and specific heat constant over the temperature range, (5) Negligible convection.

PROPERTIES: *Table A-1*, Tungsten (2900 K); $\rho = 19{,}300\ \mathrm{kg/m^3}$, $c_p \approx 185\ \mathrm{J/kg \cdot K}$.

ANALYSIS: (a) The total emissivity at Ts = 2900 K follows from Eq. 12.36 using Table 12.1 for the band emission factors,

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda,b}(T_s)d\lambda = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2(1 - F_{0\to 2\mu m}) \quad (1)$$

$$\varepsilon = 0.45 \times 0.72 + 0.1\,(1 - 0.72) = 0.352 \qquad <$$

where $F_{(0\to 2\mu m)} = 0.72$ at $\lambda T = 2\mu m \times 2900\ K = 5800\ \mu m{\cdot}K$.

(b) Perform an energy balance on the filament at the instant of time at which the current is switched off,

$$\dot{E}_{in} - \dot{E}_{out} = Mc_p \frac{dT_s}{dt}$$

$$A_s\left(\alpha G_{sur} - E\right) = A_s\left(\alpha \sigma T_s^4 - \varepsilon \sigma T_s^4\right) = Mc_p\, dT_s/dt$$

and find the change in temperature with time where $A_s = \pi DL$, $M = \rho\forall$, and $\forall = (\pi D^2/4)L$,

$$\frac{dT_s}{dt} = -\frac{\pi DL\sigma(\varepsilon T_s^4 - \alpha T_{sur}^4)}{\rho\left(\pi D^2/4\right)Lc_p} = -\frac{4\sigma}{\rho c_p D}\left(\varepsilon T_s^4 - \alpha T_{sur}^4\right)$$

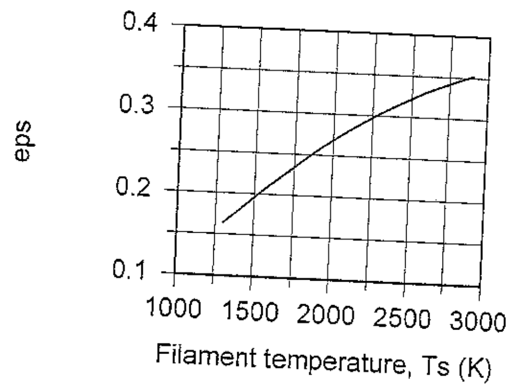
$$\frac{dT_s}{dt} = -\frac{4\times 5.67\times 10^{-8}\ W/m^2\cdot K^4(0.352\times 2900^4 - 0.1\times 300^4)K^4}{19{,}300\ kg/m^2 \times 185\ J/kg\cdot K\times 0.0008m} = -1977\ K/s \qquad <$$

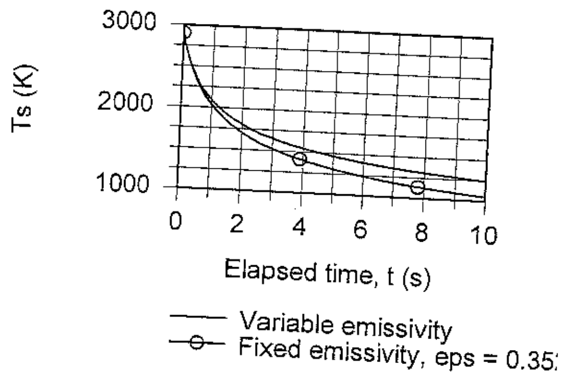
(c) Using the *IHT Tool, Radiation, Band Emission Factor,* and Eq. (1), a model was developed to calculate and plot ε as a function of T_s. See plot below.

Continued...

(d) Using the IHT *Lumped Capacitance Model* along with the IHT workspace for part (c) to determine ε as a function of T_s, a model was developed to predict T_s as a function of cooling time. The results are shown below for the variable emissivity case (ε vs. T_s as per the plot below left) and the case where the emissivity is fixed at $\varepsilon(2900\ K) = 0.352$. For the variable and fixed emissivity cases, the times to reach $T_s = 1300$ K are

$t_{var} = 8.3$ s $\qquad t_{fix} = 5.1$ s $\qquad <$





COMMENTS: (1) From the ε vs. T_s plot, note that ε increases as T_s increases. Could you have surmised as much by looking at the spectral emissivity distribution, ε_λ vs. λ?

(2) How do you explain the result that $t_{var} > t_{fix}$?

(3) The absorptivity is $\alpha = 0.1$. This is from Section 12.5.1. The results are insensitive to the absorptivity since $T_{sur} \ll T_s$.

11) At a wavelength of 4 μm, what is the temperature of the blackbody that will give an emissive power equal to 10^3 W/(m^2. μm)?

Given $\lambda = 4 \mu m$ and $E_{b\lambda}(T) = 10^3$ W/m^2 μm, and using the spectral blackbody emissive power expression, we solve for the temperature T as follows:

$$10^3 = \frac{3.743 \times 10^8}{4^5 \{\exp[1.4387 \times 10^4/(4T)] - 1\}}$$

or

$$\frac{1.4387 \times 10^4}{4T} = \ln 366.53$$

and

$$T = 609 \text{ K}$$

12) Consider a blackbody emitting at 1500 K. Determine the wavelength at which the blackbody spectral emissive power $E_{b\lambda}$ is maximum.

Give $T = 1500\,K$, from Wien's displacement law, we have

$(\lambda T)_{max} = 2897.6\,\mu m\,K$

or

$$\lambda = \frac{2897.6}{1500} = 1.93\,\mu m$$

13) There is a hole of radius 0.2 cm in large spherical enclosure whose inner surface is maintained at 800 K. Determine the rate of emission of radiative energy through this opening.

Given $T = 800K$, the blackbody emissive power can be calculated from the Stefan-Boltzmann law:

$$E_b(T) = \sigma T^4$$

$$= (5.67 \times 10^{-8})(800^4) = 23{,}224.3 \ W/m^2$$

The hole area is $A = \pi (0.002)^2 m^2$. Therefore the rate of emission of radiation energy Q through the hole becomes

$$Q = A E_b(T)$$

$$= \pi (0.002)^2 (23{,}224.3) = 0.29 \ W$$

14) A tungsten filament is heated to 2500 K. What fraction of this energy is in the visible range?

The visible range of the spectrum is $\lambda_1 = 0.4\,\mu m$ to $\lambda_2 = 0.7\,\mu m$. Given $T = 2500\,K$, for $\lambda_1 T = (0.4)(2500) = 1000\,\mu m.K^4$ we obtain from Table 12.1

$$F_{0-\lambda_1} = 0.00032$$

and for $\lambda_2 T = (0.7)(2500) = 1750\,\mu m.K^4$, we obtain

$$f_{0-\lambda_2} = 0.03392$$

Then

$$F_{0-\lambda_2} - F_{0-\lambda_1} = 0.03392 - 0.00032 = 0.0336$$

Therefore 3.36 percent of the energy is in the visible range.

15) The spectral emissivity of an opaque surface at 1000 K is given by

$\varepsilon_1=0.1$	for $\lambda_o = 0$ to $\lambda_1 = 0.5\,\mu m$
$\varepsilon_2=0.5$	for $\lambda_1 = 0.5$ to $\lambda_2 = 6\,\mu m$
$\varepsilon_3=0.7$	for $\lambda_2 = 6$ to $\lambda_3 = 15\,\mu m$
$\varepsilon_4=0.8$	for $\lambda_3 > 15\,\mu m$

Determine the average emissivity over the entire range of wavelengths and the radiation flux emitted by the material at 1000 K.

The spectral distrubition of emissivity is given in stepwise variations. Therefore, we break the integral into parts:

$$\varepsilon = \frac{\int_0^{\infty} \varepsilon_\lambda E_{\lambda b}(T)\,d\lambda}{e_b(T)} = \varepsilon_1 \int_0^{\lambda_1} \frac{E_{b\lambda}(T)}{E_b(T)} d\lambda + \varepsilon_2 \int_{\lambda_1}^{\lambda_2} \frac{E_{b\lambda}(T)}{E_b(T)} d\lambda$$

$$+ \varepsilon_3 \int_{\lambda_2}^{\lambda_3} \frac{E_{b\lambda}(T)}{E_b(T)} d\lambda + \varepsilon_4 \int_{\lambda_3}^{\infty} \frac{E_{b\lambda}(T)}{E_b(T)} d\lambda$$

$$= \varepsilon_1 f_{0-\lambda_1} + \varepsilon_2 (f_{0-\lambda_2} - f_{0-\lambda_1}) + \varepsilon_3 (f_{0-\lambda_3} - f_{0-\lambda_2}) + \varepsilon_4 (f_{0-\infty} - f_{0-\lambda_3})$$

where $f_{0-\lambda}$ is given in Table 12-1. We have

$$\lambda_1 = 0.5 \Rightarrow \lambda_1 T = (0.5)(1000) = 500\,\mu m.K \quad f_{0-\lambda_1} \cong 0$$
$$\lambda_2 = 6 \Rightarrow \lambda_2 T = (6)(1000) = 6000\,\mu m.K \quad f_{0-\lambda_2} \cong 0.73777$$
$$\lambda_3 = 15 \Rightarrow \lambda_3 T = (15)(1000) = 15000\,\mu m.K \quad f_{0-\lambda_3} = 0.96892$$

Then

$$\varepsilon = (0.1)(0) + (0.5)(0.73777 - 0) + 0.7(0.96892 - 0.73777)$$
$$+ 0.8(1 - 0.96892)$$
$$= 0.5556$$

Knowing the average emissivity $\varepsilon(T)$, the radiation energy $q(T)$ emitted by the surface per unit area at a temperature

$T = 1000K$;

$q(T) = \varepsilon(T)\sigma T^4$

$= (0.5556)(5.67 \times 10^{-8})(1000)^4$

$= 31502.5\ W/m^2$