Examples for Chapter I

1) Calculate the rate of heat flow through a 0.5 m wide, 0.3 m high and 3 mm thick steel plate, having a thermal conductivity of k = 45 W/mK when the temperature of surface at x = 0 is maintained at a constant temperature of 198°C and its temperature at x = 3 mm is 199.7°C.

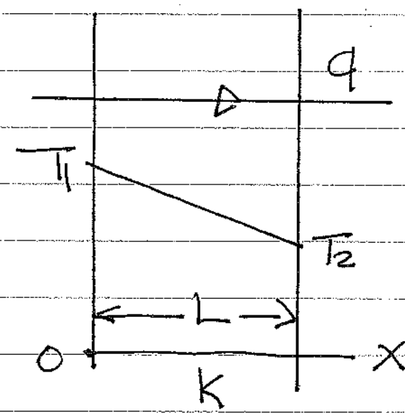
Solution:

$$q = \frac{kA(T_1 - T_2)}{L} = \frac{(45\ W/m.K)(0.5m)(0.3m)(198°C - 199.7°C)}{(0.003m)}$$

$$\Rightarrow q = -3.825\ kW$$

1) Determine the heat flow across a plane wall of 10 cm thickness with a constant thermal conductivity of 8.5 W/mK when surface temperatures are steady at 100°C and 30°C. The wall area 3 m². Also find the temperature gradient in the flow direction.

solution



$T_1 = 100°C$
$T_2 = 30°C$
$L = 10cm = 0.1m$
$K = 8.5\ W/mK$

$A = 3\ m^2$

$$q = \frac{T_1 - T_2}{\left(\frac{L}{Ak}\right)}$$

a) $q = \dfrac{100 - 30}{\dfrac{0.1}{3 \times 8.5}} = 17850\ W$

b) $q = -kA\dfrac{dT}{dx}$

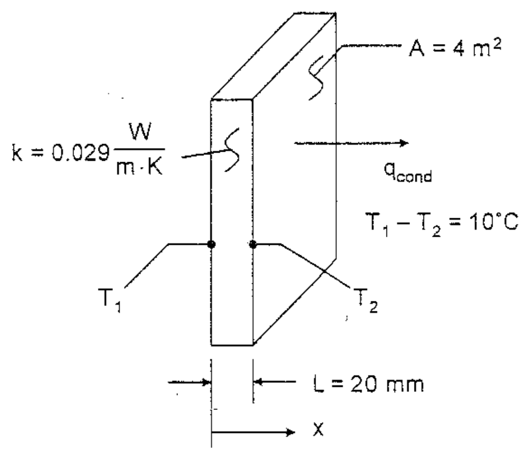
$$\frac{dT}{dx} = q/(-kA) = -17850/(8.5 \times 3) = 700°C/m$$

2) The thermal conductivity of a sheet of rigid, extruded insulation is reported to be $k = 0.029\ W/m.K$. The measured temperature difference across a 20-mm-thick sheet of the material is T_1-T_2=10°C.
   a) What is the heat flux through a 2 m × 2 m sheet of the insulation?
   b) What is the rate of heat transfer through the sheet of insulation?

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation.

FIND: (a) The heat flux through a 2 m × 2 m sheet of the insulation, and (b) The heat rate through the sheet.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Equation 1.2 the heat flux is

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L}$$

Solving,

$$q''_x = 0.029\frac{W}{m \cdot K} \times \frac{10\ K}{0.02\ m}$$

$$q''_x = 14.5\frac{W}{m^2}$$ <

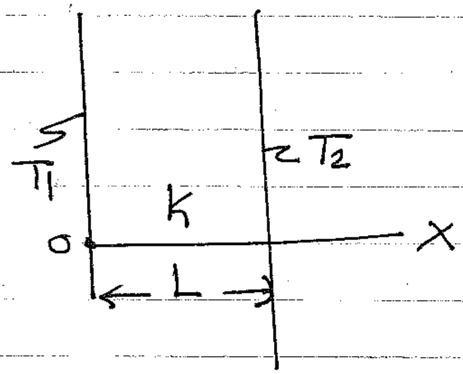
The heat rate is

$$q_x = q''_x \cdot A = 14.5\frac{W}{m^2} \times 4\ m^2 = 58\ W$$ <

COMMENTS: (1) Be sure to keep in mind the important distinction between the heat flux (W/m^2) and the heat rate (W). (2) The direction of heat flow is from hot to cold. (3) Note that a temperature διφφερενχε may be expressed in kelvins or degrees Celsius.

Example

The thermal conductivity of a plane wall



varies as

$$k = k_o(1 + bT + cT^2)$$

Determine the heat flux through the wall.

solution

$$q'' = \frac{q}{A} = -k\frac{dT}{dx}$$

$$q'' = -k_o(1 + bT + cT^2)\frac{dT}{dx}$$

or

$$\int_0^L q''\,dx = -\int_{T_1}^{T_2} k_o(1 + bT + cT^2)\,dT$$

Using Mathematica for integration

see next page

In[12]:= $-\int_{T_1}^{T_2} k_0 * \left(1 + b * T + c * T^2\right) \, dT$

Out[12]= $k_0\, T_1 + \frac{1}{2}\, b\, k_0\, T_1^2 + \frac{1}{3}\, c\, k_0\, T_1^3 - k_0\, T_2 - \frac{1}{2}\, b\, k_0\, T_2^2 - \frac{1}{3}\, c\, k_0\, T_2^3$

In[13]:= **%12 / L**

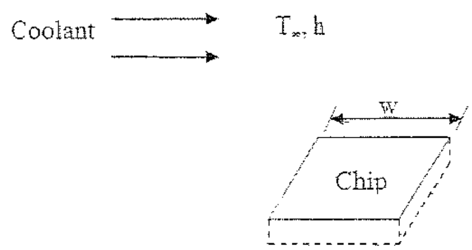
Out[13]= $\frac{k_0\, T_1 + \frac{1}{2}\, b\, k_0\, T_1^2 + \frac{1}{3}\, c\, k_0\, T_1^3 - k_0\, T_2 - \frac{1}{2}\, b\, k_0\, T_2^2 - \frac{1}{3}\, c\, k_0\, T_2^3}{L}$

In[16]:= **Factor[%13]**

Out[16]= $\frac{k_0\, (T_1 - T_2)\, \left(6 + 3\, b\, T_1 + 2\, c\, T_1^2 + 3\, b\, T_2 + 2\, c\, T_1\, T_2 + 2\, c\, T_2^2\right)}{6\, L}$

$$q'' = \frac{k_0 (T_1 - T_2)}{L} \left\{ 1 + \frac{b}{2} (T_1 + T_2) + \frac{c}{3} \left(T_1^2 + T_1 T_2 + T_2^2 \right) \right\}$$

3) A square isothermal chip is of width w = 5 mm on a side and is mounted in a substrate such that its side and back surfaces are well insulated, while the front surface is exposed to the flow of a coolant at T_∞ = 15°C. From reliability considerations, the chip temperature must not exceed T = 85°C.

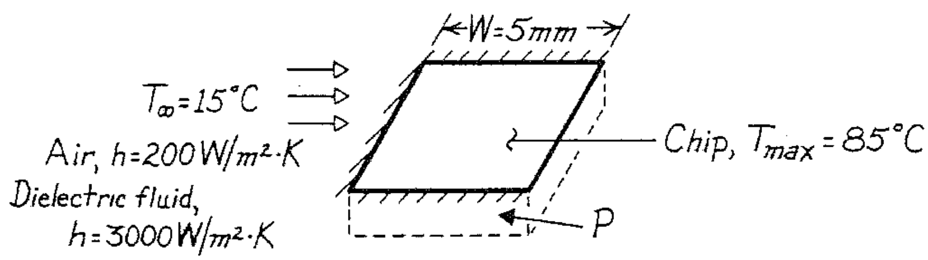


If the coolant is air and the corresponding convection coefficient is h = 200 $W/m^2.K$, what is the maximum allowable chip power? If the coolant is a dielectric liquid for which h = 3000 $W/m^2.K$, what is the maximum allowable power?

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

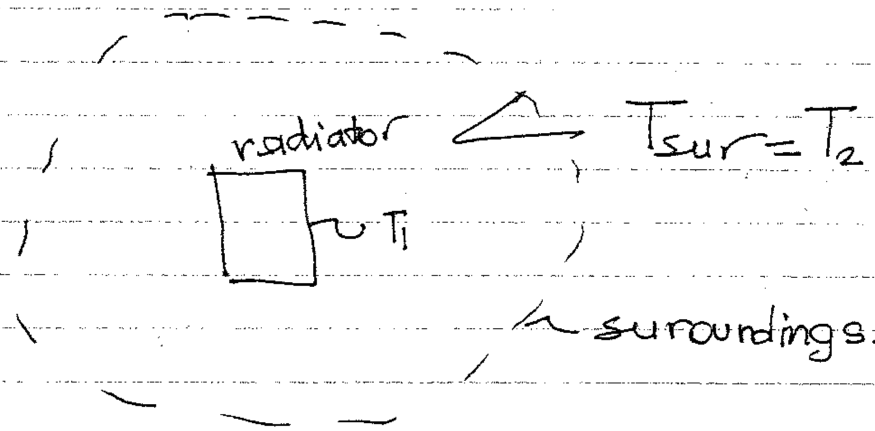
$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

$$P_{max} = 200\ W/m^2 \cdot K(0.005\ m)^2(85 - 15)\ °C = 0.35\ W. \qquad <$$

$$P_{max} = 3000\ W/m^2 \cdot K(0.005\ m)^2(85\text{-}15)\ °C = 5.25\ W. \qquad <$$

COMMENTS: Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

4) The surface temperature of a central heating radiator is 60°C. What is the net blackbody radiative heat transfer between the radiator and surroundings which are at 20°C?



$$q''_{1-2} = q''_r = \sigma (T_1^4 - T_2^4)$$

Assume surroundings and radiator behave as a black body

$T_1 = 333.15 \text{ K}$ $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

$T_2 = 293.15 \text{ K}$

$q''_{12} = 280 \text{ W/m}^2$

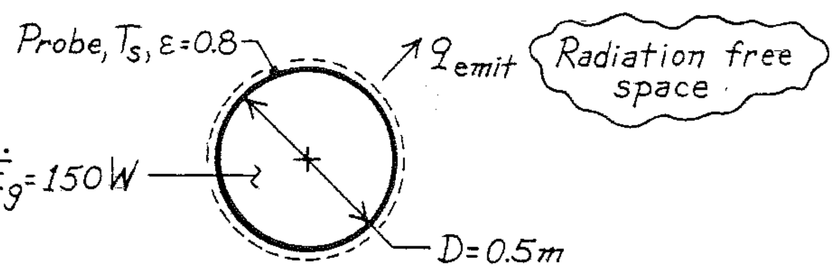
Note that a blackbody absorbs all incident radiation.

5) A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

KNOWN: Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

FIND: Probe surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

ANALYSIS: Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_s = \left(\frac{\dot{E}_g}{\varepsilon \pi D^2 \sigma} \right)^{1/4}$$

$$T_s = \left(\frac{150\,\text{W}}{0.8\pi (0.5\text{ m})^2 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4} \right)^{1/4}$$

$$T_s = 254.7\text{ K.}$$ <

COMMENTS: Incident radiation, as, for example, from the sun, would increase the surface temperature.

6) A hot plate is exposed to environment at 20°C. The measured data in boundary layer is given below.

y (m)	T (K)
0	400
0.00050	393.28
0.00102	386.30
0.00156	379.06
0.00212	371.59
0.00270	363.94
0.00331	356.17
0.00394	348.39
0.00460	340.75
0.00529	333.40
0.00600	326.54
0.00674	320.33
0.00751	314.95
0.00831	310.49
0.00914	306.98
0.01001	304.38
0.01091	302.57
0.01184	301.41
0.01282	300.71

The thermal conductivity of the environment fluid is k = 0.03 W/m.K. Calculate the local heat transfer coefficient at this particular point. Data is collected 0.5 m from the leading edge of the plate.

Mathematica 7.0 is used in the solution of the problem.

```
In[1]:= SetDirectory["F:\\A_TUTORIALS\\MATHEMATICA 7 "]

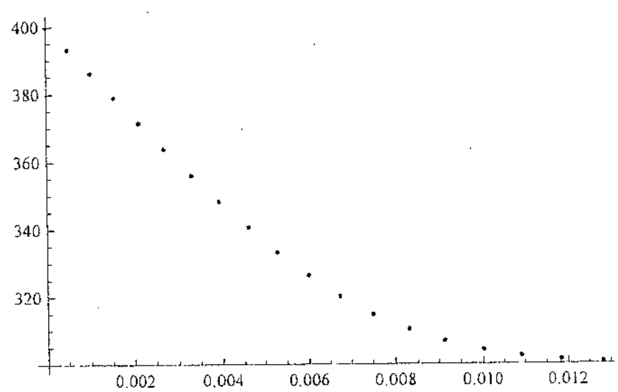
Out[1]= F:\A_TUTORIALS\MATHEMATICA 7

In[3]:= data1 = ReadList["data.dat", {Number, Number}]

Out[3]= {{0, 400}, {0.0005, 393.28}, {0.00102, 386.3}, {0.00156, 379.06}, {0.00212, 371.59},
 {0.0027, 363.94}, {0.00331, 356.17}, {0.00394, 348.39}, {0.0046, 340.75}, {0.00529, 333.4},
 {0.006, 326.54}, {0.00674, 320.33}, {0.00751, 314.95}, {0.00831, 310.49}, {0.00914, 306.98},
 {0.01001, 304.38}, {0.01091, 302.57}, {0.01184, 301.41}, {0.01282, 300.71}}

In[4]:= ListPlot[data1]

Out[4]=
```



In[34]:=

```
Temp[y_] = Fit[data1, {1, y, y^2}, y]
```

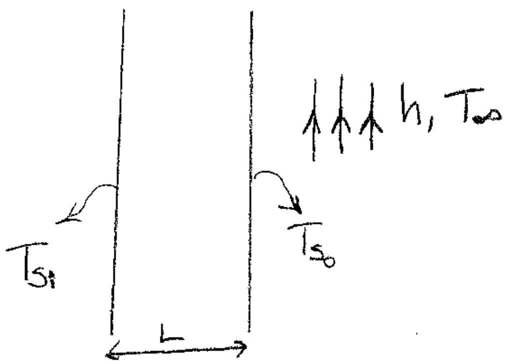
Out[34]= $402.183 - 16478.2\,y + 667905.\,y^2$

let $T = Temp(y)$

$$h = \frac{-k \left.\frac{\partial T}{\partial y}\right)_{y=0}}{T_s - T_\infty} = \frac{-(0.03)(-16478.2)}{400 - 293}$$

$$= 4.63 \text{ W/m K}$$

7) A refrigerator stands in a room where air temperature is 20°C. The surface temperature on the outside of refrigerator is 16°C. The sides are 30 mm thick and have thermal conductivity of 0.1 W/mK. The heat transfer coefficient on the outside is 10 W/m^{2}K. Assuming one dimensional conduction through sides, find surface temperature inside.



$$q = hA(T_{so} - T_{si}) \Rightarrow q'' = h(T_{so} - T_\infty)$$

$$\Rightarrow q'' = \left(10\ \frac{W}{m^2 K}\right)(16^\circ C - 20^\circ C) \Rightarrow q'' = -40\ W/m^2$$

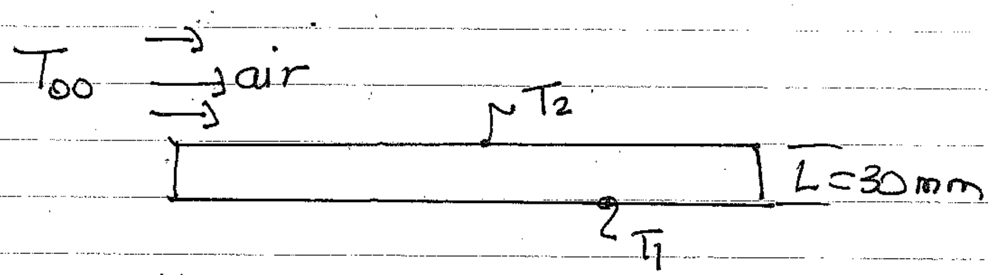
$$q'' = \frac{-k(T_{so} - T_{si})}{L} \Rightarrow T_{si} = T_{so} + \frac{q'' L}{k}$$

$$\Rightarrow T_{si} = 16^\circ C + \left[\frac{(-40\ W/m^2)(0.03\ m)}{(0.1\ W/m.K)}\right]$$

$$\Rightarrow T_{si} = 4^\circ C$$

Example

A horizontal plate ($k = 30\ W/mK$) 600 mm by 900 mm by 30 mm is maintained at 300°C. The air at 30°C flows over plate. If the convection coefficient of air over the plate is 22 W/m^2K and 250 W of heat is lost from the plate



by radiation, calculate the bottom surface temperature of the plate.

solution

$$q_k'' = q_c'' + q_r''$$

$$\frac{KA}{L}(T_1 - T_2) = \bar{h} A (T_2 - T_\infty) + 250$$

$$\frac{(30)(0.54)(T_1 - 300)}{0.03} = (22)(0.54)(300 - 30) + 250$$

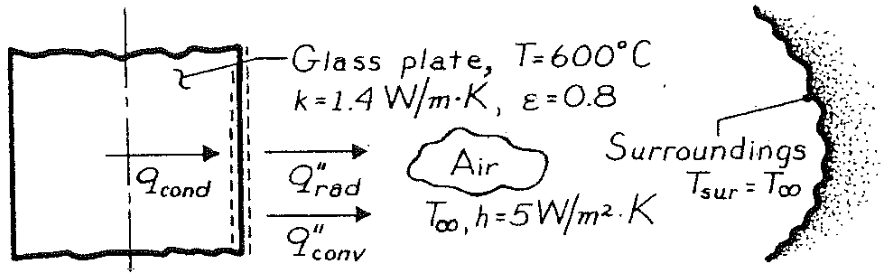
$$T_1 = 306.4\,°C$$

8) During its manufacture, plate glass at 600°C is cooled by passing air over its surface such that the convection heat transfer coefficient is h = 5 W/m^2.K. To prevent cracking, it is known that the temperature gradient must not exceed 15°C/mm at any point in the glass during the cooling process. If the thermal conductivity of the glass is 1.4 W/m.K. and its surface emissivity is 0.8, what is the lowest temperature of the air that can initially be used for the cooling? Assume that the temperature of the air equals that of the surroundings?

KNOWN: Conditions associated with surface cooling of plate glass which is initially at 600°C. Maximum allowable temperature gradient in the glass.

FIND: Lowest allowable air temperature, T_∞

SCHEMATIC:



ASSUMPTIONS: (1) Surface of glass exchanges radiation with large surroundings at $T_{sur} = T_\infty$, (2) One-dimensional conduction in the x-direction.

ANALYSIS: The maximum temperature gradient will exist at the surface of the glass and at the instant that cooling is initiated. From the surface energy balance, Eq. 1.12, and the rate equations, Eqs. 1.1, 1.3a and 1.7, it follows that

$$-k\frac{dT}{dx} - h\left(T_s - T_\infty\right) - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0$$

or, with $(dT/dx)_{max} = -15°C/mm = -15{,}000°C/m$ and $T_{sur} = T_\infty$,

$$-1.4\frac{W}{m\cdot K}\left[-15{,}000\frac{°C}{m}\right] = 5\frac{W}{m^2\cdot K}\left(873 - T_\infty\right)K$$

$$+0.8\times 5.67\times 10^{-8}\frac{W}{m^2\cdot K^4}\left[873^4 - T_\infty^4\right]K^4.$$

$$21000 = 5(873 - T_\infty) + 4.536\times 10^{-8}(873^4 - T_\infty^4)$$

Define : let $x = T_\infty$

$$f = -21000 + 5(873 - x) + 4.536\times 10^{-8}(873^4 - x^4)$$

```
f[x_] = -21000 + 5 * (873 - x) + 4.536 * (10^-8) * (873^4 - x^4)
```

$-21000 + 5\,(873 - x) + 4.536 \times 10^{-8}\,(580840612641 - x^4)$

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Plot[f[x], {x, 0, 700}]
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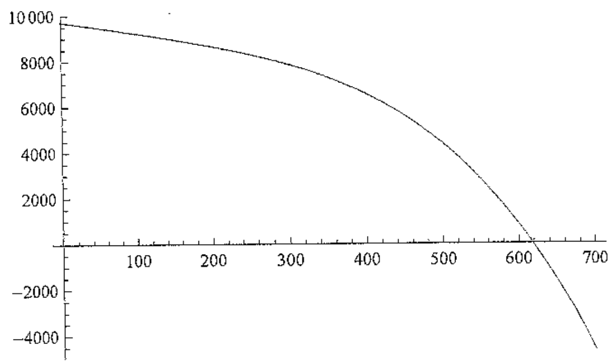


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FindRoot[f[x], {x, 600}]
```

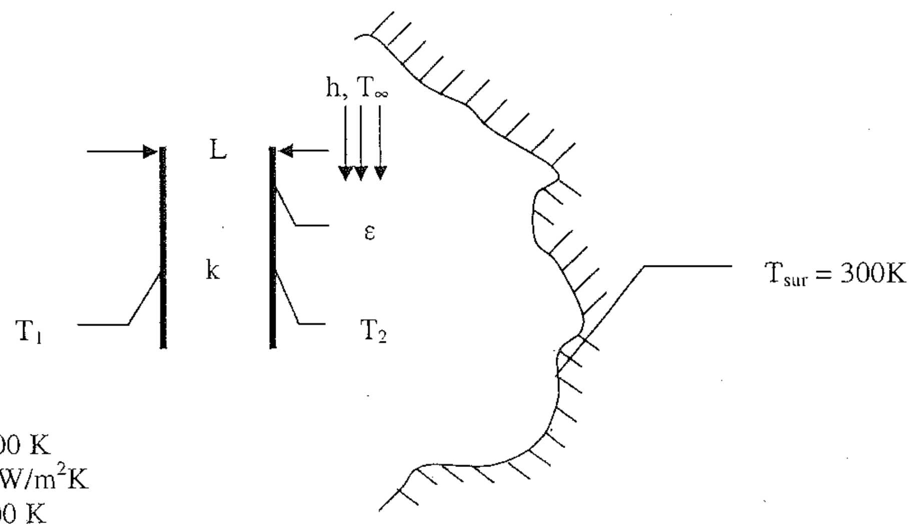
{x → 618.114}

Allowable Temperature is 618 K

COMMENTS: (1) Initially, cooling is determined primarily by radiation effects.

(2) For fixed T_∞, the surface τεμ περατυρε γραδιεντ would δεχρεασε with ινχρεασινγ time into the cooling process. Accordingly, T_∞ could be decreasing with increasing time and still keep within the maximum allowable temperature gradient.

9) Consider a thick plate as shown below. What is the temperature of the left side of the slab?

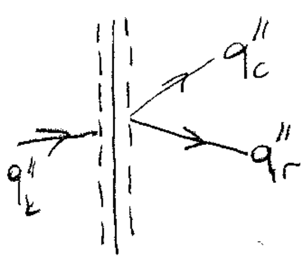


T_∞ = 300 K
h = 20 W/m²K
T_2 = 400 K
ε = 0.8
L = 0.05 m
k = 0.7 W/mK

Assumption:

1) 1-Dimensional
2) Steady-State
3) Radiation exchange with large surroundings
4) ρ, C_p, k, h constant
5) T_1 and T_2 are uniform.

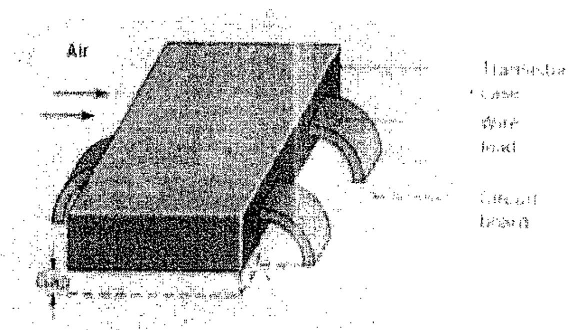
$$A\frac{k}{L}(T_1-T_2) = \underbrace{hA(T_2-T_\infty)}_{2000W} + \underbrace{\varepsilon\sigma A(T_2^4-T_{sur}^4)}_{794W}$$



$\Rightarrow T_1 = 600K$

10) Consider a surface mount type transistor on a circuit board whose temperature is maintained at 35°C. Air at 25°C flows over the upper surface of dimension 4 mm by 8 mm with a convection coefficient of 50 W/m^{2}K. Three wire loads (leads), each of cross section 1 mm by 0.25 mm and length 4 mm, conduct heat from the case to the board. The gap between the case and the board is 0.2 mm. Assuming the case is isothermal and neglecting radiation, estimate the case temperature when 150 m.W is dissipated by the board and;

a) stagnant air
b) conductive past(e) fills the gap. The thermal conductivities of the wire leads, air, and conductive paste are 25, 0.0263 and 0.12 W/mK respectively.

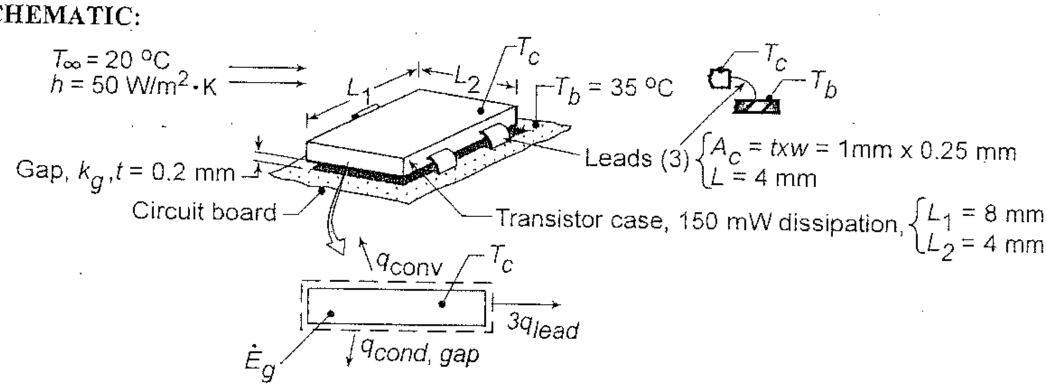


T_c – case temperature
T_b – board temperature
T_a – air temperature

KNOWN: Surface-mount transistor with prescribed dissipation and convection cooling conditions.

FIND: (a) Case temperature for mounting arrangement with air-gap and conductive paste between case and circuit board, (b) Consider options for increasing $\dot{E}_g$, subject to the constraint that $T_c = 40°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Transistor case is isothermal, (3) Upper surface experiences convection; negligible losses from edges, (4) Leads provide conduction path between case and board, (5) Negligible radiation, (6) Negligible energy generation in leads due to current flow, (7) Negligible convection from surface of leads.

PROPERTIES: (Given): Air, $k_{g,a} = 0.0263$ W/m·K; Paste, $k_{g,p} = 0.12$ W/m·K; Metal leads, k_ℓ = 25 W/m·K.

ANALYSIS: (a) Define the transistor as the system and identify modes of heat transfer.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \Delta\dot{E}_{st} = 0$$

$$-q_{conv} - q_{cond,gap} - 3q_{lead} + \dot{E}_g = 0$$

$$-hA_s(T_c - T_\infty) - k_g A_s \frac{T_c - T_b}{t} - 3k_\ell A_c \frac{T_c - T_b}{L} + \dot{E}_g = 0$$

where $A_s = L_1 \times L_2 = 4 \times 8\ \text{mm}^2 = 32 \times 10^{-6}\ \text{m}^2$ and $A_c = t \times w = 0.25 \times 1\ \text{mm}^2 = 25 \times 10^{-8}\ \text{m}^2$.

Rearranging and solving for T_c,

$$T_c = \left\{hA_sT_\infty + \left[k_gA_s/t + 3(k_\ell A_c/L)\right]T_b + \dot{E}_g\right\} / \left[hA_s + k_gA_s/t + 3(k_\ell A_c/L)\right]$$

Substituting numerical values, with the *air-gap condition* ($k_{g,a} = 0.0263$ W/m·K)

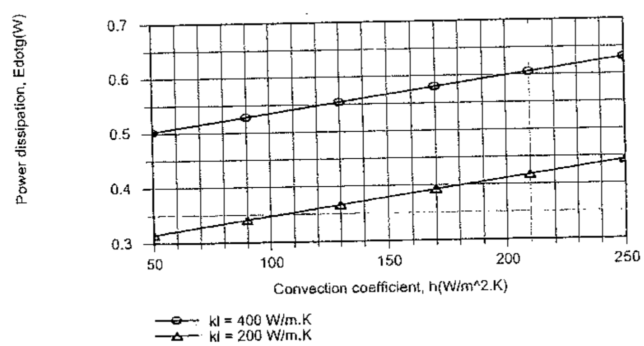
$$T_c = \left\{50\,\text{W/m}^2\cdot\text{K} \times 32\times10^{-6}\text{m}^2 \times 20^\circ\text{C} + \left[\left(0.0263\,\text{W/m}\cdot\text{K} \times 32\times10^{-6}\text{m}^2/0.2\times10^{-3}\text{m}\right)\right.\right.$$

$$\left.\left. +3\left(25\ \text{W/m}\cdot\text{K} \times 25\times10^{-8}\text{m}^2/4\times10^{-3}\text{m}\right)\right]35^\circ\text{C}\right\} / \left[1.600\times10^{-3} + 4.208\times10^{-3} + 4.688\times10^{-3}\right]\text{W/K}$$

$$T_c = 47.0^\circ\text{C}.$$

With the *paste condition* ($k_{g,p} = 0.12$ W/m·K), $T_c = 39.9°C$. As expected, the effect of the conductive paste is to improve the coupling between the circuit board and the case. Hence, T_c decreases.

(b) Using the keyboard to enter model equations into the workspace, IHT has been used to perform the desired calculations. For values of $k_\ell = 200$ and 400 W/m·K and convection coefficients in the range from 50 to 250 W/m²·K, the energy balance equation may be used to compute the power dissipation for a maximum allowable case temperature of 40°C.

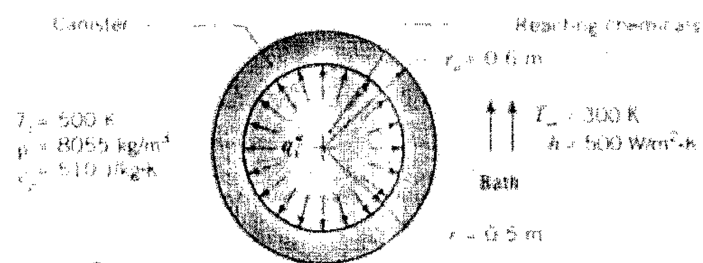


As indicated by the energy balance, the power dissipation increases linearly with increasing h, as well as with increasing k_ℓ. For $h = 250$ W/m²·K (enhanced air cooling) and $k_\ell = 400$ W/m·K (copper leads), the transistor may dissipate up to 0.63 W.

COMMENTS: Additional benefits may be derived by increasing heat transfer across the gap separating the case from the board, perhaps by inserting a highly conductive material in the gap.

11) A spherical, stainless steel (AISI 302) canister is used to store reacting chemicals that provide for a uniform heat flux q_i'' to its inner surface. The canister is suddenly submerged in a liquid bath of temperature $T_\infty < T_i$, where T_i is the initial temperature of the canister wall.

a) Assuming negligible temperature gradients in the canister wall and a constant heat flux q_i'', develop and equation that governs the variation the wall temperature with time during the transient process. What is the initial rate of change of wall temperature, if $q_i'' = 10^5\ \mathrm{W/m^2}$?
b) What is the steady-state temperature of the wall?
c) The convection coefficient depends on the velocity associated with fluid flow over the canister and whether or not the wall temperature is large enough to induce boiling in the liquid. Compute and plot the steady-state temperature as a function of h for the range $100 \le h \le 10000\ \mathrm{W/m^2K}$. Is there a value of h below which operation would be unacceptable?



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$\dot{E}_{st} = \rho c \forall \frac{dT}{dt}$$

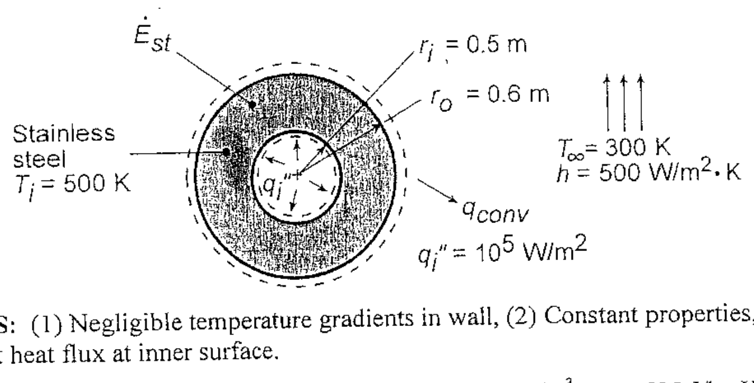
$\forall$ – volume

$$\Rightarrow \rho c \forall \frac{dT}{dt} = \dot{E}_{in} - \dot{E}_{out}$$

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: $\rho = 8055\ \mathrm{kg/m^3}$, $c_p = 535\ \mathrm{J/kg \cdot K}$.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho\frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5\frac{W}{m^2}(0.5m)^2 - 500\frac{W}{m^2\cdot K}(0.6m)^2(500-300)K\right]}{8055\frac{kg}{m^3}510\frac{J}{kg\cdot K}\left[(0.6)^3 - (0.5)^3\right]m^3}$$

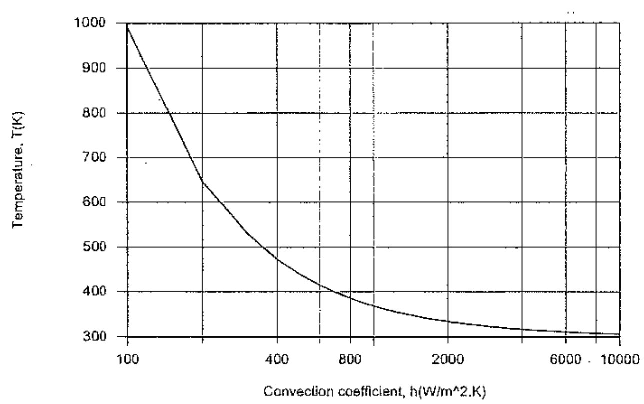
$$\left.\frac{dT}{dt}\right)_i = -0.084\,K/s.$$ <

(b) Under steady-state conditions with $\dot{E}_{st} = 0$, it follows that

$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300K + \frac{10^5\,W/m^2}{500\,W/m^2\cdot K}\left(\frac{0.5m}{0.6m}\right)^2 = 439K$$ <

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of h < 1000 $W/m^2\cdot K$. For T > 380 K, boiling will occur at the canister surface, and for T > 410 K a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T.



Although the canister remains well below the melting point of stainless steel for h = 100 $W/m^2\cdot K$, boiling should be avoided, in which case the convection coefficient should be maintained at h > 1000 $W/m^2\cdot K$.

COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$, $S \equiv 3q_i'' r_i^2 / \rho c_p\left(r_o^3 - r_i^3\right)$, $R = 3hr_o^2/\rho c_p\left(r_o^3 - r_i^3\right)$. Note results for $t \to \infty$ and for $S = 0$.