Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer

CHAPTER 6
EXAMPLE SOLUTIONS

1. In a flow over surface, velocity and temperature profiles are of the forms:

$$u(y) = Ay + By^2 - Cy^3 \quad (m/s)$$
$$T(y) = D + Ey + Fy^2 - Gy^3 \quad (K)$$

where coefficients A through G are constants. Obtain expressions for the friction coefficient C_f and the convection coefficient h in terms of U_∞, T_∞ and approximate profile coefficients and fluid properties.

2. Air at a free stream temperature of $T_\infty = 20^oC$ is parallel flow over a flat plate of length L = 5m and temperature $T_s = 90^oC$. However, obstacles placed in the flow intensify mixing with increasing distance x from the leading edge, and the spatial variation of temperatures measured in the boundary layer is correlated by an expression of the form:

$$T(x,y) = 20 + 70e^{-600xy} \quad (^oC)$$

where x and y are in meters. Determine and plot the manner in which the local convection coefficient h varies with x. Evaluate the average convection coefficient $\bar{h}$ for the plate.

3. During air cooling of potatoes, the heat transfer coefficient, for combined convection, radiation, and evaporation is determined experimentally to be as shown:

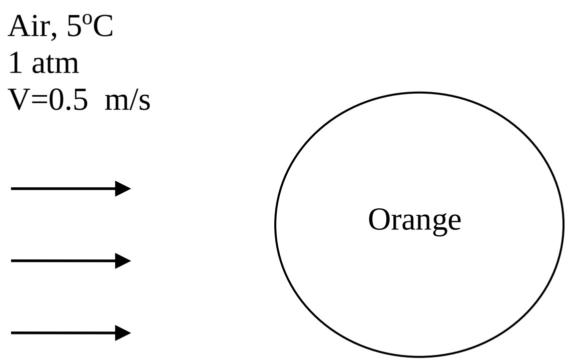
Air velocity, m/s	Heat Transfer Coefficient, $W/m^2.^oC$
0.66	14.0
1.0	19.1
1.36	20.2
1.73	24.4

Consider a 10 cm diameter potato initially at 20°C with a thermal conductivity of 0.49 W/m.°C. Potatoes are cooled by refrigerated air at 5°C at a velocity of 1 m/s. Determine the initial rate of heat transfer from a potato, and the initial value of the temperature gradient in the potato at the surface.

4. During air cooling of oranges, the heat transfer coefficient for combined convection, radiation, and evaporation for air velocities of $0.11<V<0.6$ m/s is determined experimentally and is expressed as

$$h=5.05k_{air}Re^{1/3}/D,$$

where the diameter D is the characteristic length. Oranges are cooled by refrigerated air at 5°C and 1 atm at a velocity of 0.5 m/s. Determine (a) the initial rate of heat transfer from a 7 cm diameter orange initially at 15°C with a thermal conductivity of 0.50 W/m.°C, (b) the value of the initial temperature gradient inside the orange at the surface, and (c) the value of the Nusselt number.



5. Will a thermal boundary layer develop in flow over a surface even if both the fluid and the surface are at the same temperature?

6. A 4 m×4 m flat plate maintained at constant temperature of 80°C is subjected to parallel flow of air at 1 atm, 20°C, and 10 m/s. The total drag force acting on the upper surface of the plate is measured to be 2.4 N. Using momentum heat transfer analogy, determine the average convection heat transfer coefficient, and the rate of heat transfer between the upper surface of the plate and the air.

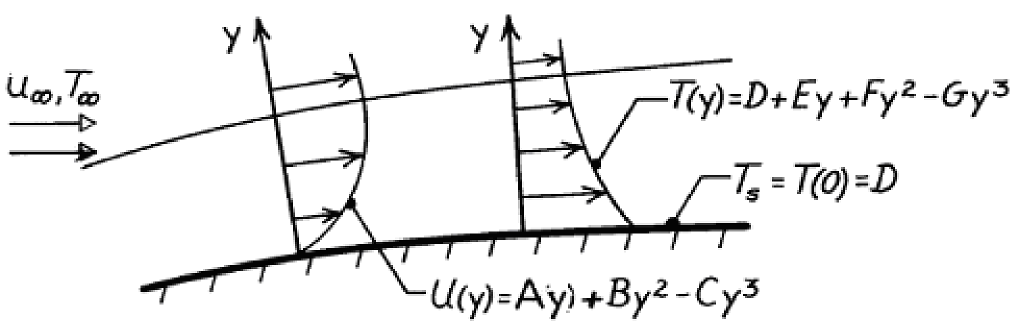
SOLUTIONS

1)

KNOWN: Form of the velocity and temperature profiles for flow over a surface.

FIND: Expressions for the friction and convection coefficients.

SCHEMATIC:



ANALYSIS: The shear stress at the wall is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right]_{y=0} = \mu \left[A + 2By - 3Cy^2\right]_{y=0} = A\mu.$$

Hence, the friction coefficient has the form,

$$C_f = \frac{\tau_s}{\rho u_\infty^2 / 2} = \frac{2A\mu}{\rho u_\infty^2}$$

$$C_f = \frac{2A\nu}{u_\infty^2}. \qquad <$$

The convection coefficient is

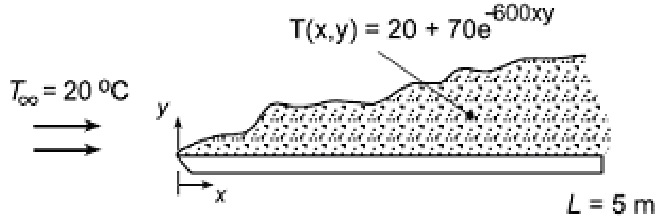
$$h = \frac{-k(\partial T/\partial y)_{y=0}}{T_s - T_\infty} = \frac{-k\left[E + 2Fy - 3Gy^2\right]_{y=0}}{D - T_\infty}$$

$$h = \frac{-k\,E}{D - T_\infty}. \qquad <$$

2)

FIND: Variation of local convection coefficient along the plate and value of average coefficient.

SCHEMATIC:



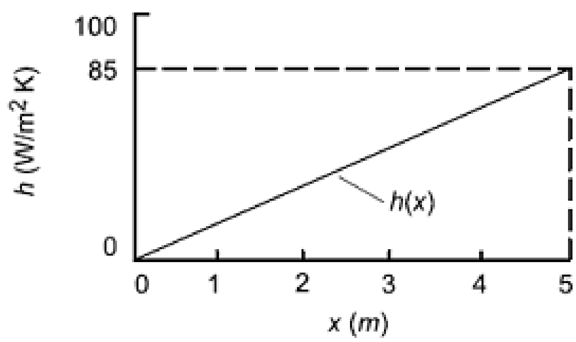
ANALYSIS:

$$h = -\frac{k\,\partial T/\partial y|_{y=0}}{(T_s - T_\infty)} = +\frac{k(70\times 600x)}{(T_s - T_\infty)}$$

where $T_s = T(x,0) = 90°C$. Evaluating k at the arithmetic mean of the freestream and surface temperatures, $\overline{T} = (20 + 90)°C/2 = 55°C = 328$ K, Table yields $k = 0.0284$ W/m·K. Hence, with $T_s - T_\infty = 70°C = 70$ K,

$$h = \frac{0.0284\,W/m\cdot K(42,000x)K/m}{70K} = 17x\left(W/m^2\cdot K\right)$$

and the convection coefficient increases linearly with x.



The average coefficient over the range $0 \le x \le 5$ m is

$$\overline{h} = \frac{1}{L}\int_0^L h\,dx = \frac{17}{5}\int_0^5 x\,dx = \frac{17}{5}\frac{x^2}{2}\Big|_0^5 = 42.5\,W/m^2\cdot K$$

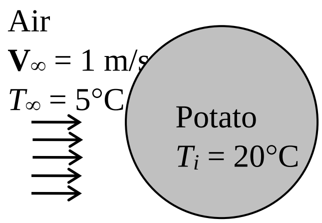
3) Heat transfer coefficients at different air velocities are given during air cooling of potatoes. The initial rate of heat transfer from a potato and the temperature gradient at the potato surface are to be determined.

Assumptions **1** steady operating condition exists. **2** Potato is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface.

Properties The thermal conductivity of the potato is given to be $k = 0.49$ W/m.°C.

Analysis The initial rate of heat transfer from a potato is

$$A_s = \pi D^2 = \pi(0.10\text{ m})^2 = 0.03142\text{ m}^2$$



$$\dot{Q} = hA_s(T_s - T_\infty) = (19.1\,\mathrm{W/m^2.°C})(0.03142\ \mathrm{m^2})(20-5)°\mathrm{C} = \mathbf{9.0\,W}$$

where the heat transfer coefficient is obtained from the table at 1 m/s velocity. The initial value of the temperature gradient at the potato surface is

$$\dot{q}_{\mathrm{conv}} = \dot{q}_{\mathrm{cond}} = -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(19.1\,\mathrm{W/m^2.°C})(20-5)°\mathrm{C}}{(0.49\ \mathrm{W/m.°C})} = \mathbf{-585°C/m}$$

4) The expression for the heat transfer coefficient for air cooling of some fruits is given. The initial rate of heat transfer from an orange, the temperature gradient at the orange surface, and the value of the Nusselt number are to be determined.

Assumptions **1** steady operating condition exists. **2** Orange is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Properties of water is used for orange.

Properties

k = 0.50 W/m.°C orange

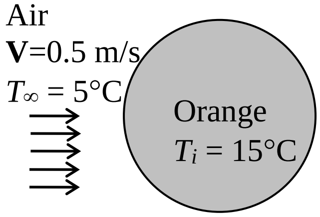
At the film temperature of $(T_s + T_\infty)/2$ = (15+5)/2 = 10°C ,we obtain air properties

For air:

$k = 0.02439$ W/m.°C,

$\upsilon = 1.426\times10^{-5}\ \mathrm{m^2/s}$

Analysis (*a*) The Reynolds number, the heat transfer coefficient, and the initial rate of heat transfer from an orange are



$$A_s = \pi D^2 = \pi(0.07\ \text{m})^2 = 0.01539\ \text{m}^2$$

$$\text{Re} = \frac{\mathbf{V}_\infty D}{\upsilon} = \frac{(0.5\ \text{m/s})(0.07\ \text{m})}{1.426\times10^{-5}\ \text{m}^2/\text{s}} = 2454$$

$$h = \frac{5.05 k_{air}\ \text{Re}^{1/3}}{D} = \frac{5.05(0.02439\ \text{W/m.°C})(2454)^{1/3}}{0.07\ \text{m}} = 23.73\ \text{W/m}^2.\text{°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (23.73\ \text{W/m}^2.\text{°C})(0.01539\ \text{m}^2)(15-5)\text{°C} = \mathbf{3.65\ W}$$

(*b*) The temperature gradient at the orange surface is determined from

$$\dot{q}_{cond} = \dot{q}_{conv}$$

$$-k_{orange}\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k_{orange}} = -\frac{(23.73\ \text{W/m}^2.\text{°C})(15-5)\text{°C}}{(0.50\ \text{W/m.°C})} = \mathbf{-475\ °C/m}$$

(*c*) The Nusselt number is $\text{Re} = \frac{hD}{k} = \frac{(23.73\ \text{W/m}^2.\text{°C})(0.07\ \text{m})}{0.02439\ \text{W/m.°C}} = \mathbf{68.11}$

5) A thermal boundary layer will not develop in flow over a surface if both the fluid and the surface are at the same temperature since there will be no heat transfer in that case.

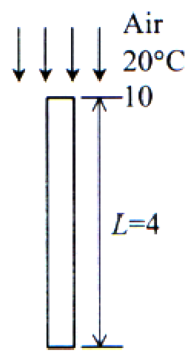
6)

A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection heat transfer coefficient and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 20°C and 1 atm are (Table A-15)

$$\rho = 1.204 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg-K}, \quad \text{Pr} = 0.7309$$



Analysis The flow is along the 4-m side of the plate, and thus the characteristic length is L = 4 m. Both sides of the plate is exposed to air flow, and thus the total surface area is

$$A_s = 2WL = 2(4\text{ m})(4\text{ m}) = 32\text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \longrightarrow C_f = \frac{F_f}{\rho A_s V^2/2} = \frac{2.4\text{ N}}{(1.204\text{ kg/m}^3)(32\text{ m}^2)(10\text{ m/s})^2/2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 0.006229$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho V C_p}{\text{Pr}^{2/3}} = \frac{0.006229}{2}\frac{(1.204\text{ kg/m}^3)(10\text{ m/s})(1007\text{ J/kg}\cdot°\text{C})}{(0.7309)^{2/3}} = \mathbf{46.54\ W/m^2\cdot C}$$

Them the rate of heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (46.54\text{ W/m}^2\cdot°\text{C})(32\text{ m}^2)(80-20)°\text{C} = \mathbf{89{,}356\ W}$$