Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer

Chapter 9
Free Convection Examples

1. A large vertical plate 4.0 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer if the plate is 10 m wide.

2. A horizontal pipe 1 ft (0.3048 m) in diameter is maintained at temperature of 250°C in a room where the ambient air is at 15 °C. Calculate the free convection heat loss per meter of length.

3. A fine wire having a diameter of 0.02 mm is maintained at a constant temperature of 54°C by an electric current. The wire is exposed to air at 1 at m and 0°C. Calculate the electric power necessary to maintain the wire temperature if the length is 50 cm.

4. A flat, isothermal heating panel, 0.5 m wide by 1m high, is mounted to one wall of a large room. The surface of the panel has an emissivity of 0.90 and is maintained at 400 K. If the walls and air in the room are at 300 K, what is the net rate at which heat is transferred from the panel to the room?

5. In a plant location near a furnace a net radiant energy flux of 800 W/m² is incident on a vertical metal surface 3,5m high and 2 m wide. The metal is insulated on the back side and painted black so that all the incoming radiation is lost by free convection to surrounding air at 30°C. What average temperature will be attained by the plate?

6. P 9.35

7. P 9.92

8. P 9.86

9. P 9.97

from textbook

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Chapter 9
Example Solutions

1. A large vertical plate 4.0 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer if the plate is 10 m wide.

We first determine the film temperature as;

$$T_f = \frac{60+10}{2} = 35°C = 308\ K$$

The properties of interest are thus:

$$\beta = \frac{1}{308} = 3.25\times10^{-3}\ K^{-1} \qquad k = 0.02685\ W/mK$$

$$\nu = 16.5\times10^{-6}\ m^2/s \qquad Pr = 0.7$$

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}Pr = \frac{(9.8)(3.25\times10^{-3})(60-10)(4)^3}{(16.5\times10^{-6})^2}(0.7)$$

$$\Rightarrow Ra_L = 2.62\times10^{11}$$

We then obtain $\overline{Nu}_L$:

$$\overline{Nu}_L = \left[0.825 + \frac{0.387\,Ra_L^{1/6}}{[1+(0.492/Pr)^{9/16}]^{8/27}}\right]^2$$

$$\Rightarrow \overline{Nu}_L = \left[0.825 + \frac{(0.387)(2.62\times10^{11})^{1/6}}{[1+(0.492/0.7)^{9/16}]^{8/27}}\right] \Rightarrow \overline{Nu}_L = 716$$

~~The heat transfer coefficient is then:~~

$$\bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{(716)(0.02685)}{(4.0)} \Rightarrow \bar{h} = 4.80\ W/m^2 °C$$

The heat transfer is : $q = \bar{h} A (T_s - T_\infty) = (4.80)(4)(10)(60-10) = 9606\ W$

2. A horizontal pipe 1 ft (0.3048 m) in diameter is maintained at temperature of 250°C in a room where the ambient air is at 15°C. Calculate the free convection heat loss per meter of length.

We first determine the Rayleigh number and then select the approciate constants from Table 9-1. The properties of air are evaluated at the film temperature

$$T_f = \frac{T_s + T_\infty}{2} = \frac{250+15}{2} = 132.5\,°C = 405.5\,K$$

$k = 0.03406\ W/m°C$

$\nu = 26.54 \times 10^{-6}\ m^2/s$

$\beta = \frac{1}{T_f} = \frac{1}{405.5} = 2.47 \times 10^{-3}\ K^{-1}$

$Pr = 0.687$

$$Ra_D = Gr_D Pr = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} Pr = \frac{(9.8)(2.47\times10^{-3})(250-15)(0.3048)^3(0.687)}{(26.54\times10^{-6})^2}$$

$\Rightarrow Ra_D = 1.571 \times 10^8$

$\overline{Nu_D}$ is defined as: $\overline{Nu_D} = C Ra_D^n$

From Table 9.1, $C = 0.125$, $n = 0.333$ so that:

$$\overline{Nu_D} = (0.125)(1.571\times10^8)^{0.333} = 67.03$$

$$\overline{Nu_D} = \frac{\bar{h}D}{k} \Rightarrow \bar{h} = \frac{k\overline{Nu_D}}{D} = \frac{(0.03406)(67.03)}{(0.3048)} = 7.49\ W/m^2°C$$

The heat transfer per unit length is then calculated from:

$$\frac{q}{L} = \bar{h}\pi D(T_s - T_\infty) = (7.49)\pi(0.3048)(250-15) = 1685.4\ W/m = 1.69\ kW/m$$

3. A fine wire having a diameter of 0.02 mm is maintained at a constant temperature of 54 °C by an electric current. The wire is exposed to air at 1 atm and 0 °C. Calculate the electric power necessary to maintain the wire temperature if the length is 50 cm.

The film temperature is $T_f = (54+0)/2 = 27\,^\circ C = 300\,K$,

So the properties are:

$\beta = 1/300 = 0.00333\ K^{-1}$ $\quad\quad \nu = 15.69 \times 10^{-6}\ m^2/s$

$k = 0.02624\ W/m\,^\circ C$ $\quad\quad Pr = 0.708$

The Rayleigh number is calculated as:

$$Ra_D = Gr_D\,Pr = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\,Pr$$

$$\Rightarrow Ra_D = \frac{(9.8)(0.00333)(54-0)(0.02\times10^{-3})^3}{(15.69\times10^{-6})^2}(0.708) = 4.05\times10^{-5}$$

$\overline{Nu}_D$ is defined as:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,Ra_D^n$$

From Table 9.1 we find $C = 0.675$ and $n = 0.058$ so that:

$$\overline{Nu}_D = (0.675)(4.05\times10^{-5})^{0.058} \Rightarrow \overline{Nu}_D = 0.375$$

$$\Rightarrow \bar{h} = \overline{Nu}_D\left(\frac{k}{D}\right) = \frac{(0.375)(0.02624)}{(0.02\times10^{-3})} \Rightarrow \bar{h} = 492.6\ W/m^2\,^\circ C$$

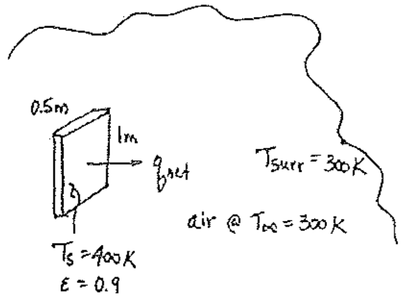
The heat transfer or power required is then:

$$q = \bar{h}A(T_s - T_\infty) = (492.6)\,\pi(0.02\times10^{-3})(0.5)(54-0)$$

$$\Rightarrow q = 0.836\ W$$

Example - 4

A flat, isothermal heating panel, 0.5 m wide by 1 m high, is mounted to one wall of a large room. The surface of the panel has an emissivity of 0.90 and is maintained at 400 K. If the walls and air in the room are at 300 K, what is the net rate at which heat is transferred from the panel to the room?

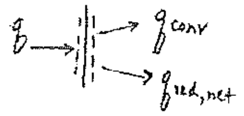


assume:

1-D transfer from panel (isothermal)
neglect conduction through panel
constant properties of air @ $T_f = 350\,K$
large enclosure (radiation)
panel is a gray surface ($\alpha = \varepsilon$)
steady state

analysis:

surface energy balance,



$$\cancel{\dot{E}_{st}} = \dot{E}_{in} + \cancel{\dot{E}_g} - \dot{E}_{out}$$

$$0 = q - (q_{conv} + q_{rad,net})$$

$$\therefore \quad q = hA_s(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{surr}^4)A_s$$

to find h: determine flow conditions

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr$$

(A.4 @ $T_f = 350K$: $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.03\ \frac{W}{m \cdot K}$, $Pr = 0.7$)

assume air is an ideal gas: $\beta = \frac{1}{T}$

$$Ra_L = \frac{(9.81\ m/s^2)\left(\frac{1}{350\ K}\right)(400-300)K\ (1m)^3}{(20.92 \times 10^{-6})^2}(0.7) = 4.483 \times 10^9$$

$Ra_L > Ra_c \quad \therefore$ turbulent flow

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{(0.387)(4.483\times10^9)^{1/6}}{\left[1+(0.492/0.7)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{(0.387)(4.483\times10^9)^{1/6}}{\left[1+(0.492/0.7)^{9/16}\right]^{8/27}}\right\}^2 = 195.6$$

$$\bar{h} = (195.6)(0.03\ W/m\cdot K)/(1m) = 5.87\ W/m^2\cdot K$$

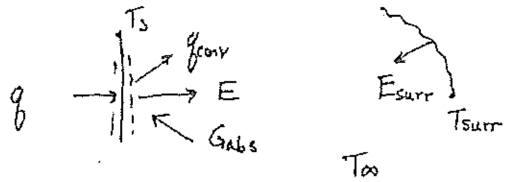
$$q = \Big[\underbrace{(5.87\ \tfrac{W}{m^2\cdot K})(400-300)K}_{q''_{conv} = 586.7\ W/m^2} + \underbrace{(0.9)(5.67\times10^{-8}\ \tfrac{W}{m^2K^4})(400^4-300^4)K}_{q''_{rad} = 893.0\ W/m^2}\Big]\underbrace{(1m)(0.5m)}_{A_s}$$

$$\therefore\quad q = 740\ W$$

Comments:

Often radiation is comparable to free convection, if not more significant.

aside:



$$E = \varepsilon\sigma T_s^4$$

$$G_{abs} = \alpha G = \alpha\sigma T_{surr}^4$$

if surface is "gray" : $\alpha = \varepsilon$

$$q = q_{conv} + E - G_{abs} = q_{conv} + \underbrace{\varepsilon\sigma T_s^4 - \varepsilon\sigma T_{surr}^4}_{q_{rad,net}}$$

Example-5

In a plant location near a furnace, a net radiant energy flux of 800 W/m^2 is incident on a vertical metal surface 3.5 m high and 2 m wide. The metal is insulated on the back side and painted black so that all the incoming radiation is lost by free convection to the surrounding air at 30°C. What average temperature will be attained by the plate?

Solution

We treat this problem as one with constant heat flux on the surface. Since we do not know the surface temperature, we must make an estimate for determining T_f and the air properties. An *approximate* value of h for free-convection problems is 10 W/m^2 · °C, and so, *approximately*,

$$\Delta T = \frac{q_w}{h} \approx \frac{800}{10} = 80°\text{C}$$

Then

$$T_f \approx \frac{80}{2} + 30 = 70°\text{C} = 343\ \text{K}$$

At 70°C the properties of air are

$$\nu = 2.043 \times 10^{-5}\ \text{m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = 2.92 \times 10^{-3}\ \text{K}^{-1}$$

$$k = 0.0295\ \text{W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7$$

From Equation , with $x = 3.5$ m,

$$\text{Gr}_x^* = \frac{g\beta q_w x^4}{k\nu^2} = \frac{(9.8)(2.92 \times 10^{-3})(800)(3.5)^4}{(0.0295)(2.043 \times 10^{-5})^2} = 2.79 \times 10^{14}$$

We may therefore use Equation to evaluate h_x:

$$\begin{aligned} h_x &= \frac{k}{x}(0.17)(\text{Gr}_x^* \, \text{Pr})^{1/4} \\ &= \frac{0.0295}{3.5}(0.17)(2.79 \times 10^{14} \times 0.7)^{1/4} \\ &= 5.36\ \text{W/m}^2 \cdot °\text{C}\ [0.944\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}] \end{aligned}$$

In the turbulent heat transfer governed by Equation (, we note that

$$\text{Nu}_x = \frac{hx}{k} \sim (\text{Gr}_x^*)^{1/4} \sim (x^4)^{1/4}$$

or h_x does not vary with x, and we may take this as the average value. The value of $h = 5.41$ W/m^2 · °C is less than the approximate value we used to estimate T_f. Recalculating ΔT, we obtain

$$\Delta T = \frac{q_w}{h} = \frac{800}{5.36} = 149°\text{C}$$

Our new film temperature would be

$$T_f = 30 + \frac{149}{2} = 104.5°\text{C}$$

At 104.5°C the properties of air are

$$\nu = 2.354 \times 10^{-5}\ \text{m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = 2.65 \times 10^{-3}/\text{K}$$

$$k = 0.0320\ \text{W/m} \cdot °\text{C} \qquad \text{Pr} = 0.695$$

Then

$$\mathrm{Gr}_x^* = \frac{(9.8)(2.65\times10^{-3})(800)(3.5)^4}{(0.0320)(2.354\times10^{-5})^2} = 1.75\times10^{14}$$

and h_x is calculated from

$$\begin{aligned} h_x &= \frac{k}{x}(0.17)(\mathrm{Gr}_x^*\,\mathrm{Pr})^{1/4} \\ &= \frac{(0.0320)(0.17)}{3.5}[(1.758\times10^{14})(0.695)]^{1/4} \\ &= 5.17\ \mathrm{W/m^2\cdot{}^\circ C}\ [-0.91\ \mathrm{Btu/h\cdot ft^2\cdot{}^\circ F}] \end{aligned}$$

Our new temperature difference is calculated as

$$\Delta T = (T_w - T_\infty)_{\mathrm{av}} = \frac{q_w}{h} = \frac{800}{5.17} = 155^\circ\mathrm{C}$$

The average wall temperature is therefore

$$T_{w,\mathrm{av}} = 155 + 30 = 185^\circ\mathrm{C}$$

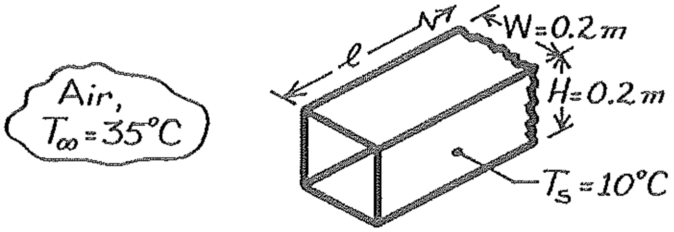
Another iteration on the value of T_f is not warranted by the improved accuracy that would result.

PROBLEM 9.35

KNOWN: Surface temperature of a long duct and ambient air temperature.

FIND: Heat gain to the duct per unit length of the duct.

SCHEMATIC:



ASSUMPTIONS: (1) Surface radiation effects are negligible, (2) Ambient air is quiescent.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 \approx 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $k = 0.0263\ W/m{\cdot}K$, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, $Pr = 0.707$, $\beta = 1/T_f$.

ANALYSIS: The heat gain to the duct can be expressed as

$$q' = 2q'_s + q'_t + q'_b = \left(2\bar{h}_s \cdot H + \bar{h}_t \cdot W + \bar{h}_b \cdot W\right)\left(T_\infty - T_s\right). \qquad (1)$$

Consider now correlations to estimate $\bar{h}_s, \bar{h}_t$, and $\bar{h}_b$. From Eq. 9.25, for the sides with $L \equiv H$,

$$Ra_L = \frac{g\beta\left(T_\infty - T_s\right)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\left(1/300K\right)\left(35-10\right)K \times \left(0.2m\right)^3}{15.89\times10^{-6}m^2/s \times 22.5\times10^{-6}m^2/s} = 1.827\times10^7. \quad (2)$$

Eq. 9.27 is appropriate to estimate $\bar{h}_s$,

$$\overline{Nu}_L = 0.68 + \frac{0.670\,Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.827\times10^7\right)^{1/4}}{\left[1+\left(0.492/0.707\right)^{9/16}\right]^{4/9}} = 34.29$$

$$\bar{h}_s = \overline{Nu}_L \cdot k/L = 34.29 \times 0.0263\,W/m\cdot K/0.2m = 4.51\,W/m^2\cdot K. \qquad (3)$$

For the top and bottom portions of the duct, $L \equiv A_s/P \approx W/2$, (see Eq. 9.29), find the Rayleigh number from Eq. (2) with $L = 0.1$ m, $Ra_L = 2.284 \times 10^6$. From the correlations, Eqs. 9.30 and 9.32 for the top and bottom surfaces, respectively, find

$$\bar{h}_t = \frac{k}{\left(W/2\right)} \times 0.54\,Ra_L^{1/4} = \frac{0.0263\,W/m\cdot K}{0.1m} \times 0.54\left(2.284\times10^6\right)^{1/4} = 5.52\,W/m^2\cdot K. \qquad (4)$$

$$\bar{h}_b = \frac{k}{\left(W/2\right)} \times 0.52\,Ra_L^{1/5} = \frac{0.0263\,W/m\cdot K}{0.1m} \times 0.52\left(2.284\times10^6\right)^{1/5} = 2.56\,W/m^2\cdot K. \qquad (5)$$

The heat rate, Eq. (1), can now be evaluated using the heat transfer coefficients estimated from Eqs. (3), (4), and (5).

$$q' = \left(2\times4.51\,W/m^2\cdot K\times0.2m + 5.52\,W/m^2\cdot K\times0.2m + 2.56\,W/m^2\cdot K\times0.2m\right)\left(35-10\right)K$$

$$q' = 85.5\,W/m. \qquad <$$

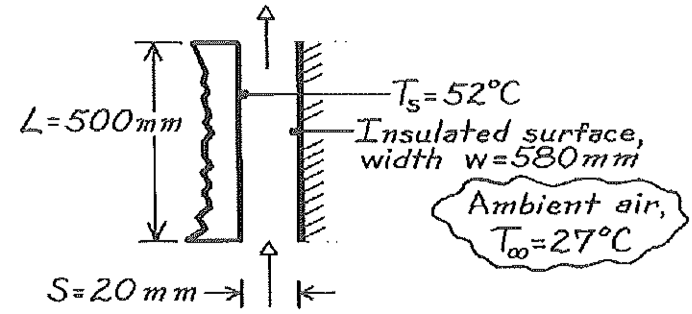
COMMENTS: Radiation surface effects will be significant in this situation. With knowledge of the duct emissivity and surroundings temperature, the radiation heat exchange could be estimated.

PROBLEM 9.82

KNOWN: Vertical air vent in front door of dishwasher with prescribed width and height. Spacing between isothermal and insulated surface of 20 mm.

FIND: (a) Heat loss from the tub surface and (b) Effect on heat rate of changing spacing by ± 10 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vent forms vertical parallel isothermal/adiabatic plates, (3) Ambient air is quiescent.

PROPERTIES: *Table A-4*, ($T_f = (T_s + T_\infty)/2 = 312.5$K, 1 atm): $\nu = 17.15 \times 10^{-6}$ m^2/s, $\alpha = 24.4 \times 10^{-6}$ m^2/s, $k = 27.2 \times 10^{-3}$ W/m·K, $\beta = 1/T_f$.

ANALYSIS: The vent arrangement forms two vertical plates, one is isothermal, T_s, and the other is adiabatic $\left(q'' = 0\right)$. The heat loss can be estimated from Eq. 9.37 with the correlation of Eq. 9.45 using $C_1 = 144$ and $C_2 = 2.87$ from Table 9.3:

$$Ra_S = \frac{g\beta\left(T_s - T_\infty\right)S^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\left(1/312.5\text{ K}\right)\left(52-27\right)\text{K}\left(0.020\,\text{m}\right)^3}{17.15\times10^{-6}\,\text{m}^2/\text{s}\times24.4\times10^{-6}\,\text{m}^2/\text{s}} = 14{,}988$$

$$q = A_s\left(T_s - T_\infty\right)\frac{k}{S}\left[\frac{C_1}{\left(Ra_S S/L\right)^2} + \frac{C_2}{\left(Ra_S S/L\right)^{1/2}}\right]^{-1/2} = \left(0.500\times0.580\right)\text{m}^2\times$$

$$\left(52-27\right)\text{K}\frac{0.0272\,\text{W/m}\cdot\text{K}}{0.020\,\text{m}}\left[\frac{C_1}{\left(Ra_S S/L\right)^2} + \frac{C_2}{\left(Ra_S S/L\right)^{1/2}}\right]^{-1/2} = 28.8\text{W}. \qquad <$$

(b) To determine the effect of the spacing at S = 30 and 10 mm, we need only repeat the above calculations with these results

S (mm)	Ra_S	q (W)	
10	1874	26.1	<
30	50,585	28.8	<

Since it would be desirable to minimize heat losses from the tub, based upon these calculations, you would recommend a decrease in the spacing.

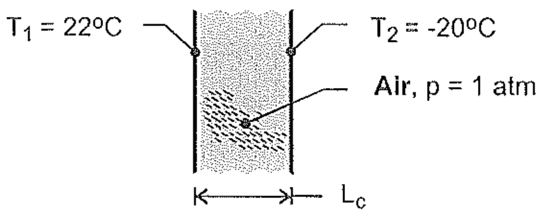
COMMENTS: For this situation, according to Table 9.3, the spacing corresponding to the maximum heat transfer rate is $S_{max} = (S_{max}/S_{opt}) \times 2.15(Ra_S/S^3L)^{-1/4} = 14.5$ mm. Find $q_{max} = 28.5$ W. Note that the heat rate is not very sensitive to spacing for these conditions.

PROBLEM 9.86

KNOWN: Critical Rayleigh number for onset of convection in vertical cavity filled with atmospheric air. Temperatures of opposing surfaces.

FIND: Maximum allowable spacing for heat transfer by conduction across the air. Effect of surface temperature and air pressure.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Rayleigh number is $Ra_{L,c} = 2000$, (2) Constant properties.

PROPERTIES: *Table A-4*, air [$T = (T_1 + T_2)/2 = 1°C = 274K$]: $\nu = 13.6 \times 10^{-6}$ m^2/s, k = 0.0242 W/m·K, $\alpha = 19.1 \times 10^{-6}$ m^2/s, $\beta = 0.00365$ K^{-1}.

ANALYSIS: With $Ra_{L,c} = g\,\beta(T_1 - T_2)\,L_c^3/\alpha\nu$,

$$L_c = \left[\frac{\alpha\nu Ra_{L,c}}{g\beta(T_1 - T_2)}\right]^{1/3} = \left[\frac{19.1 \times 13.6 \times 10^{-12}\,m^4/s^2 \times 2000}{9.8\,m/s^2 \times 0.00365\,K^{-1} \times 42°C}\right]^{1/3} = 0.007\,m = 7\,mm \qquad <$$

The critical value of the spacing, and hence the corresponding thermal resistance of the air space, increases with a decreasing temperature difference, $T_1 - T_2$, and decreasing air pressure. With $\nu = \mu/\rho$ and $\alpha \equiv k/\rho c_p$, both quantities increase with decreasing p, since ρ decreases while μ, k and c_p are approximately unchanged.

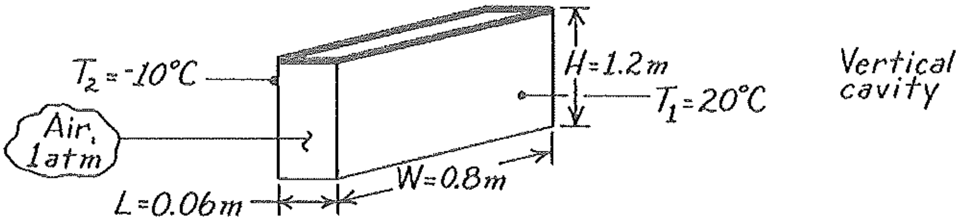
COMMENTS: (1) For the prescribed conditions and $L_c = 7$ mm, the conduction heat flux across the air space is $q'' = k(T_1 - T_2)/L_c = 0.0242\,W/m \cdot K \times 42°C/0.007m = 145\,W/m^2$, (2) With triple pane construction, the conduction heat loss could be reduced by a factor of approximately two, (3) Heat loss is also associated with radiation exchange between the panes.

PROBLEM 9.87

KNOWN: Temperatures and dimensions of a window-storm window combination.

FIND: Rate of heat loss by free convection.

SCHEMATIC:



ASSUMPTIONS: (1) Both glass plates are of uniform temperature with insulated interconnecting walls and (2) Negligible radiation exchange.

PROPERTIES: *Table A-4*, Air (278K, 1 atm): $\nu = 13.93 \times 10^{-6}\ m^2/s$, k = 0.0245 W/m·K, $\alpha = 19.6 \times 10^{-6}\ m^2/s$, Pr = 0.71, $\beta = 0.00360\ K^{-1}$.

ANALYSIS: For the vertical cavity,

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\,m/s^2\left(0.00360\,K^{-1}\right)(30°C)(0.06m)^3}{19.6\times10^{-6}m^2/s\times13.93\times10^{-6}m^2/s}$$

$$Ra_L = 8.37\times10^5.$$

With (H/L) = 20, Eq. 9.52 may be used as a first approximation for Pr = 0.71,

$$\overline{Nu}_L = 0.42\,Ra_L^{1/4}\,Pr^{0.012}\,(H/L)^{-0.3} = 0.42\left(8.37\times10^5\right)^{1/4}(0.71)^{0.012}(20)^{-0.3}$$

$$\overline{Nu}_L = 5.2$$

$$\bar{h} = \overline{Nu}_L\,\frac{k}{L} = 5.2\,\frac{0.0245\,W/m\cdot K}{0.06m} = 2.1\,W/m^2\cdot K.$$

The heat loss by free convection is then

$$q = \bar{h}\,A\left(T_1 - T_2\right)$$

$$q = 2.1\,W/m^2\cdot K\left(1.2m\times0.8m\right)(30°C) = 61\,W.$$ <

COMMENTS: In such an application, radiation losses should also be considered, and infiltration effects could render heat loss by free convection significant.