Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer

Chapter-6 Example Solutions

Example-1

The velocity profile u(x,y) for laminar boundary layer flow over a flat plate is given by

$$\frac{u(x,y)}{U_\infty} = \frac{3}{2}\left[\frac{y}{\delta(x)}\right] - \frac{1}{2}\left[\frac{y}{\delta(x)}\right]^3$$

where boundary layer thickness $\delta(x)$ is

$$\delta(x) = \sqrt{\frac{280}{13}\frac{\upsilon x}{U_\infty}}$$

Develop an expression for the local drag coefficient C_f and average drag coefficient $\bar{C}_f$ over a distance x=L from leading edge of the plate.
Solution

$$\mathbf{C_f = \frac{\tau_s}{\frac{1}{2}\rho U_\infty^2} = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{1}{2}\rho U_\infty^2} = \frac{3\upsilon}{U_\infty \delta(x)}}$$

Substituting the above expression for $\boldsymbol{\delta(x)}$ into this relation ,we obtain the local friction coefficient

$$C_f = \frac{3\upsilon}{U_\infty}\sqrt{\frac{13}{280}\frac{U_\infty}{\upsilon x}} = \sqrt{\frac{117}{280}\frac{\upsilon}{U_\infty x}} = \frac{0.664}{\sqrt{Re_x}}$$

$$C_f = \frac{0.664}{\sqrt{Re_x}}$$

The average skin friction coefficient $\bar{C}_f$ over $0 < x < L$ is defined as

$$\bar{C}_f = \frac{1}{L}\int_0^L C_f(x)dx$$

Introducing local friction coefficient into above equation we obtain the mean friction coefficient $\bar{C}_f$

$$\bar{C}_f = \frac{1}{L}\sqrt{\frac{117}{280}\frac{\nu}{U_\infty}}\int_0^L x^{-1/2}dx = \frac{1.328}{\sqrt{Re_L}}$$

$$\bar{C}_f = \frac{1.328}{\sqrt{Re_L}}$$

where Re_L is the average Reynolds number and is defined as

$$Re_L = \frac{\rho U_\infty L}{\mu}$$

Example-2

The temperature profile in a thermal boundary layer for flow over a flat plate is
is given by

$$\frac{T(x,y)-T_s}{T_\infty - T_s} = \frac{3}{2}\frac{y}{\Delta} - \frac{1}{2}\left(\frac{y}{\Delta}\right)^3$$

and the thickness of the thermal boundary layer δ *is the function of x and is given by*

$$\delta\ (x) = 4.53\ \frac{x}{Re_x^{1/2}\,Pr^{1/3}}$$

where $Pr = \frac{\mu C_p}{k}$ *and* $Re = \frac{\rho u_\infty x}{\mu}$

Develop the expressions for both the local and average heat transfer coefficients.

Solution

Given: The temperature profile in a thermal boundary layer

$$\frac{T-T_s}{T_\infty - T_s} = \frac{3}{2}\left(\frac{y}{\Delta}\right) - \frac{1}{2}\left(\frac{y}{\Delta}\right)^3$$

$$\Delta = 4.53\ \frac{x}{Re_x^{1/2}\,Pr^{1/3}} \quad \text{with} \quad Pr = \frac{\mu C_p}{k_f} \quad \text{and} \quad Re = \frac{\rho u_\infty x}{\mu}$$

To find: Expressions for local and average heat transfer coefficients

Analysis: The local heat transfer coefficient h_x is expressed as

$$h_x = \frac{k\left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_\infty - T_s}$$

For a given temperature profile

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left[\frac{3}{2}\frac{1}{\Delta} - \frac{1}{2}\times\frac{3y^2}{\Delta^3}\right]_{y=0} = \frac{3(T_\infty - T_s)}{2\Delta}$$

Then

$$h_x = \frac{3k\ (T_\infty - T_s)}{2(T_\infty - T_s)\Delta} = \frac{3}{2}\frac{k_f}{\Delta}$$

Introducing the expression for Δ we obtain

$$h_x = \frac{3}{2}\times\frac{k_f\times Re_x^{1/2}\,Pr^{1/3}}{4.53\,x} = \mathbf{0.332}\frac{k_f}{x}\,Re_x^{1/2}\ Pr^{1/3} \qquad \textbf{Answer}$$

This expression can be arranged in the dimensionless form as

$$Nu_x = \frac{h_x x}{k_f} = 0.332\ Re_x^{1/2}\ Pr^{1/3}$$

where Nu_x is called the local Nusselt number.

The average heat transfer coefficient is

$$h = \frac{1}{L}\int_0^L h_x\,dx = 0.332\,k\ \frac{\Pr^{1/3}}{L}\int_0^L \frac{1}{x}\times\sqrt{\frac{\rho u_\infty x}{\mu}}\,dx$$

$$= 0.332\ k\ \ \Pr^{1/3}\sqrt{\frac{\rho u_\infty}{\mu}}\times\frac{1}{\mathbf{L}}\int_0^L x^{-1/2}\,dx$$

$$= 0.332\ \frac{k_f}{L}\Pr^{1/3}\sqrt{\frac{\rho u_\infty}{\mu}}\left[\frac{L^{1/2}}{1/2}\right] = 2\times 0.332\frac{k_f}{L}\sqrt{\frac{\rho u_\infty L}{\mu}}\Pr^{1/3}$$

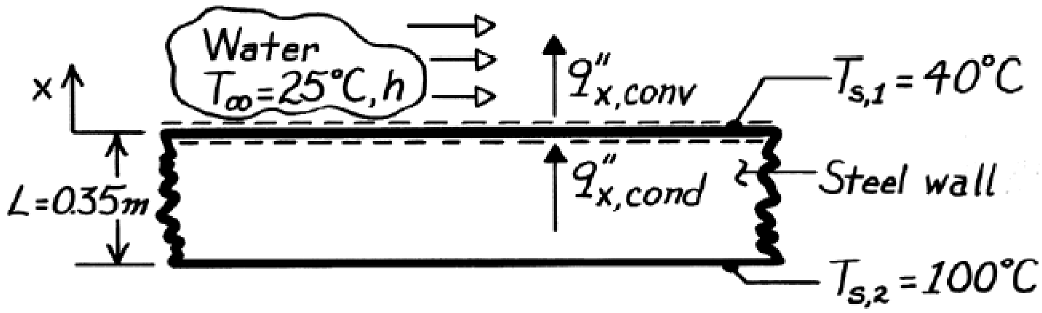
$$h = 2\times 0.332\ \frac{k_f}{\mathbf{L}}\mathrm{Re}_L^{1/2}\Pr^{1/3} = 2h_x\Big|_{x=L}$$

so the average Nusselt number Nu is **2Nu$_x$**. **Answer**

$$\overline{\mathbf{Nu}}_{\mathbf{L}} = \mathbf{0.664}\ \mathrm{Re}_L^{1/2}\Pr^{1/3}$$ **Answer**

Example 3

Water at a temperature of $T_\infty = 25\,^0C$ flows over one of the surfaces of a steel wall (AISI 1010) whose temperature is $T_{s1} = 40\,^0C$. The wall is 0.35 m thick, and its other surface temperature is $T_{s2} = 100\,^0C$. For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.



Solution

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Steel Type AISI 1010 (70°C = 343K), k_s = 61.7 W/m·K; *Table A-6*, Water (32.5°C = 305K), k_f = 0.62 W/m·K.

ANALYSIS: (a) Applying an energy balance to the control surface at x = 0, it follows that

$$q''_{x,cond} - q''_{x,conv} = 0$$

and using the appropriate rate equations,

$$k_s \frac{T_{s,2} - T_{s,1}}{L} = h\left(T_{s,1} - T_\infty\right).$$

Hence,

$$h = \frac{k_s}{L}\frac{T_{s,2} - T_{s,1}}{T_{s,1} - T_\infty} = \frac{61.7\ \text{W/m}\cdot\text{K}}{0.35\text{m}}\frac{60^\circ\text{C}}{15^\circ\text{C}} = 705\ \text{W/m}^2\cdot\text{K}. \qquad <$$

(b) The gradient in the wall at the surface is

$$\left(dT/dx\right)_s = -\frac{T_{s,2} - T_{s,1}}{L} = -\frac{60^\circ\text{C}}{0.35\text{m}} = -171.4^\circ\text{C/m}.$$

In the water at x = 0, the definition of h gives

$$\left(dT/dx\right)_{f,x=0} = -\frac{h}{k_f}\left(T_{s,1} - T_\infty\right)$$

$$\left(dT/dx\right)_{f,x=0} = -\frac{705\ \text{W/m}^2\cdot\text{K}}{0.62\ \text{W/m}\cdot\text{K}}\left(15^\circ\text{C}\right) = -17{,}056^\circ\text{C/m}. \qquad <$$