Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer

Chapter-5 Example Solutions

Fall 2015

1) A solid copper sphere of 10-cm diameter [$\rho = 8954\, kg/m^3$, $c = 383\, J/kg.^0C$, $k = 386\, W/m.^0C$] initially at a uniform temperature $T_i = 250\,^0C$, is suddenly immersed in a well-stirred fluid which is maintained at a uniform temperature $T_\infty = 50\,^0C$ .The heat transfer coefficient between the sphere and the fluid is $\bar{h} = 200\, W/m^2.^0C$
(a) Check whether the lumped system analysis is suitable
(b) If it is suitable , determine the temperature of copper sphere at t=5,10 and 20 min after the immersion.

$\rho = 8954\ kg/m^3$ $\quad T_i = 250\,°C$ $\quad T_\infty = 50\,°C$

$c = 383\ J/kg\,K$ $\quad \bar{h} = 200\ W/m^2\,°C$

$k = 386\ W/m\,°C$

$$L_c = \frac{V}{A_s} = \frac{(4/3)\pi r_o^3}{4\pi r_o^2} = \frac{r_o}{3} = \frac{D}{6} = \frac{0.1}{6} = 0.01666\ m$$

$$Bi_c = \frac{\bar{h} L_c}{k} = \frac{(200)(0.01666)}{386} = 8.6\times10^{-3}$$

$Bi_c < 0.1$ Lumped model is valid

b) $$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-mt}$$

$$m = (\bar{h} A_s / (\rho c V)) = 3.5\times10^{-3}\ s^{-1}$$

$$T(t) = T_\infty + (T_i - T_\infty) e^{-mt}$$
$$= 50 + 200\, e^{-0.0035t}$$

$t_1 = 300\,s$ $\quad T = 120\,°C$ $\quad t_2 = 600\,s$ $\quad T = 74.5\,°C$

$t_3 = 1200\,s$ $\quad T = 53\,°C$

2) A solid rod [α= $2x10^{-5}$ m^2/s and k=60 W/(m.°C)] of diameter D=6 cm, initially at temperature T_i=800 °C, is suddenly dropped into an oil bath at T_∞=50. °C. The heat transfer coefficient between the fluid and the surface is h=400 W/(m^2.°C).

(a) Using the transient-temperature charts, determine the centerline temperature 10 min after immersion in the fluid.

(b) How long will it take the centerline temperature to reach 100 °C?

Answer: (a) 54.5 °C ; (b) 5 min 47 s

$\alpha = 2\times10^{-5}\ m^2/s \quad k = 60\ w/m°C \quad D = 6cm \quad T_i = 800°C$

$T_\infty = 50°C \quad h = 400\ w/m^2°C$

a) centerline temperature after 600 s

$$F_o = \frac{\alpha t}{r_o^2} = 13.3$$

$$\frac{1}{Bi} = \frac{k}{hr_o} = \frac{60}{(400)(0.03)} = 5$$

$$\frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = 0.006$$

$$T_o = 50 + 0.006(800-50) = 54.5°C$$

b) How long it will take center temperature to reach 100°C?

$$\frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{100-50}{800-50} = 0.067$$

$$\frac{1}{Bi} = 5$$

$$F_o = \frac{\alpha t}{r_o^2}$$

$$7.7 = \frac{\alpha t}{r_o^2} = \frac{(2\times10^{-5})(t)}{(0.03)^2}$$

$$t = 347\ s$$

3) An orange of diameter 10 cm is initially at a uniform temperature of 30° C. It is placed in a refrigerator in which the air temperature is 2° C. If the transfer coefficient between the air and the surface of the orange is h=50 W/(m^2.°C), determine the time required for the center of the orange to reach 10 °C. Assume the thermal properties of the orange are the same as those of water at the same temperature. [α= 1.4×10^{-7} m^2/s and k=0.59 W/(m.°C)]
Answer: 1 h 32 min

$D = 10$ cm $\alpha = 1.4\times10^{-7}\ m^2/s$ $k = 0.59\ W/m°C$

$T_i = 30°C$ $T_\infty = 2°C$ $\bar{h} = 50\ W/m^2.°C$

How long it will take for center of orange to reach 10°C?

$r_o = 0.05\ m$

$$Bi_c = \frac{\bar{h} L_c}{k} = \frac{\bar{h} r_o}{3k} = \frac{(50)(0.05)}{3(0.59)} = 1.41$$

$Bi > 0.1$ use Heisler charts

$$\frac{1}{Bi} = \frac{k}{h r_o} = \frac{0.59}{(50)(0.05)} = 0.234$$

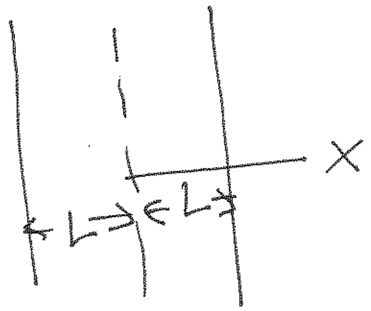
$$\frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{10-2}{30-2} = 0.286$$

$F_o = 0.31$

$$F_o = \frac{\alpha t}{r_o^2}$$

$$0.31 = \frac{(1.4\times10^{-7})t}{(0.05)^2} \Rightarrow t = 5535.7\ s$$

4) A steel plate [α= 1.2×10^{-5} m²/s and k=43 W/(m.°C)] of thickness 2L=10 cm, initially at a uniform temperature of 240 C, is suddenly immersed in an oil bath T_∞=40. °C. The convection heat transfer coefficient between the fluid and the surface is h=600 W/(m².°C). How long will it take for the center plane to cool to 100 °C? What fraction of the initial energy is removed during this time?



$\alpha = 1.2\times10^{-5}\ m^2/s \quad k = 43\ W/m\,°C$

$2L = 10\ cm$

$L = 5\ cm \quad T_i = 240°C$

$h = 600\ \frac{W}{m^2\,°C} \quad T_\infty = 40\ °C$

How long it will take for plate center temperature to cool to 100 °C

$L = \frac{0.1}{2} = 0.05\,m$

$\frac{1}{Bi} = \frac{k}{hL} = \frac{43}{(600)(0.05)} = 1.43$

$\frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{100-40}{240-40} = 0.3$

$F_o = 2.4 \qquad F_o = \frac{\alpha t}{L^2} \qquad t = \frac{F_o L^2}{\alpha}$

$t = \frac{(2.4)(0.05)^2}{(1.2\times10^{-5})} = 500\ s$

Fraction of energy removed during this period

$Q_o = \rho c (2L)(A)(T_i - T_\infty) = ? \qquad \alpha = \frac{k}{\rho c} \Rightarrow c = \frac{k}{\rho \alpha}$

$= 457.4 \frac{J}{kg\,°C}$

$= (7833)(457.4)(2\times0.05)(240-40)$

$= 72.847\ MJ$

$Bi = \frac{1}{1.43} = 0.7$

$Bi^2 F_o = (0.7)^2 (2.4) = 1.17$

$Bi = 0.7$

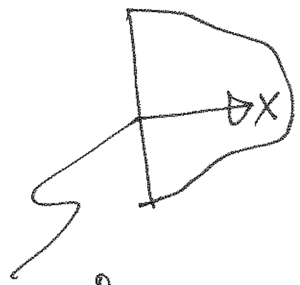
$\frac{Q}{Q_o} = 0.77$

$Q = (0.77) Q_o = 56.1\ MJ$

5) A thick concrete slab (α= 7x10^{-7} m^2/s) is initially at a uniform temperature T_i=60 ºC. One of its surfaces is suddenly lowered to 10 ºC. By treating this as a one-dimensional transient heat conduction problem in a semi-infinite medium, determine the temperatures at depths 5 and 10 cm from the surface 30 min after the surface temperature is lowered.

$\alpha = 7\times10^{-7}\ m^2/s \qquad T_i = 60^\circ C$

$t = 30\ min = 1800\ s$



$T_o = 10^\circ C$

$$\xi = \frac{x}{2\sqrt{\alpha t}} = \frac{x}{2\sqrt{7\times10^{-7}\times1800}} = 14x$$

$x = 0.05\ m \qquad \xi = 14\times0.05 = 0.7$ from figure

$$\therefore \frac{T - T_o}{T_i - T_o} = \frac{T-10}{60-10} = 0.67 \rightarrow T = (0.67)(50)+10$$
$$= 43.5^\circ C$$

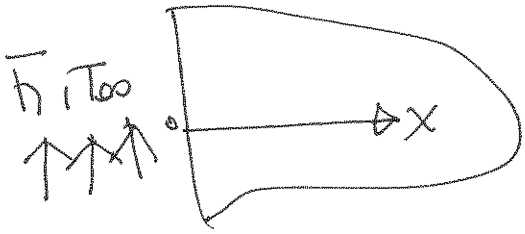
$x = 0.1\ m \qquad \xi = (14)(0.1) = 1.4$

from figure

$$\frac{T - T_o}{T_i - T_o} = 1.14$$

$$T = 10 + (60-10)(1.14) = 57.5^\circ C$$

6) A thick bronze [x= 0.86×10^{-5} m^2/s and k= 26/W(m. °C)] is initially at a uniform temperature 250 °C. Suddenly the surface is exposed to a coolant 25 °C. Assuming that the heat transfer coefficient for convection between the fluid and the surface is 150 W/(m^2.°C), determine the temperature 5 cm from the surface 10 min after the exposure.
Answer: 205 °C



$\alpha = 0.86 \times 10^{-5}\ m^2/s$

$k = 26\ \frac{W}{m\ °C}$ $\quad T_i = 250\ °C$

$T_\infty = 25\ °C$

$\bar{h} = 150\ \frac{W}{m^2\ °C}$

$t = 10\ min = 600\ s$

$$\xi = \frac{x}{2\sqrt{\alpha t}} = \frac{0.05}{2\sqrt{(0.86 \times 10^{-5})(600)}} = 0.35$$

$$\frac{\bar{h}\sqrt{\alpha t}}{k} = \frac{150\sqrt{(0.86 \times 10^{-5})(600)}}{26} = 0.41$$

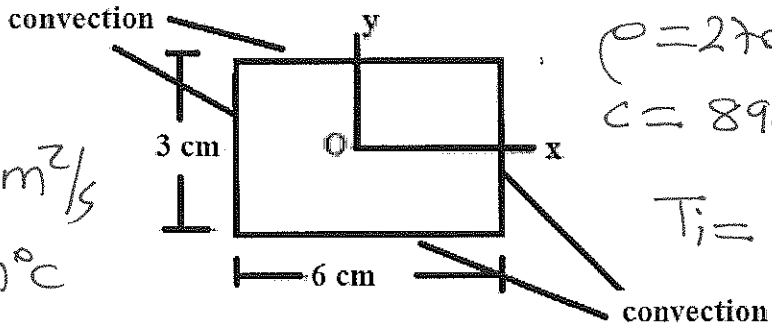
$$1 - \frac{T - T_\infty}{T_i - T_\infty} = 0.2$$

$$1 - \frac{T - T_\infty}{T_i - T_\infty} = 0.2$$

$$\frac{T - T_\infty}{T_i - T_\infty} = 1 - 0.2 = 0.8$$

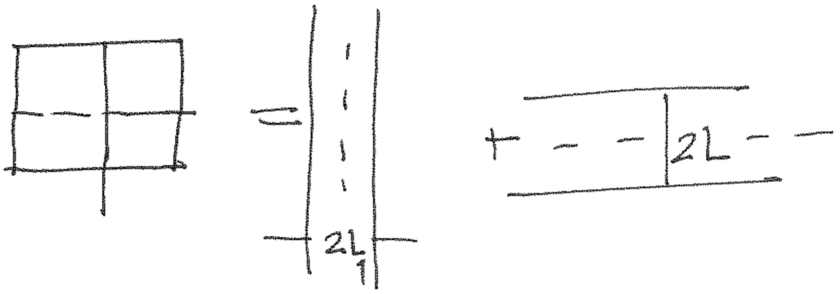
$$T = 25 + 0.8(250 - 25) = 205\ °C$$

7)) A rectangular aluminum bar 6 cm by 3 cm [k=200 W/(m.°C), c_p = 890 J/(kg.° C), ρ=2700 kg/m^3,and α= 8.4x10^{-5} m^2/s] is initially at a uniform temperature T_i=175 °C. Suddenly the surfaces are subjected to convective cooling with a heat transfer coefficient h=250 W/(m^2.°C) into an ambient at T_∞=25° C as shown in figure. Determine the center temperature T_o of the bar t=1 min after the start of the cooling.
Answer: 107.5 °C



$\rho = 2700$ kg/m^3
$c = 890$ J/kg °C
$\alpha = 8.4\times10^{-5}$ m^2/s
$k = 200$ W/m°C
$T_i = 175$ °C
$T_\infty = 25$ °C
$\bar{h} = 250 \frac{W}{m^2 °C}$
$t = 1 min = 60 s$

We have two slabs here



First slab

$$L_1 = \frac{6}{2} = 3cm = 0.03m$$

$$(Fo)_1 = \frac{\alpha t}{L_1^2} = \frac{(8.4\times10^{-5})(60)}{(0.03)^2} = 5.6$$

$$\frac{1}{Bi} = \frac{k}{\bar{h}L_1} = \frac{200}{(250)(0.03)} = 26.7$$

$$\frac{\theta_o}{\theta_i} = 0.85$$

$$\therefore \left(\frac{\theta_o}{\theta_i}\right)_1 = 0.85 \text{ for first slab}$$

Second slab:

$$L_2 = \frac{3}{2} = 1.5cm = 0.015m$$

$$(Fo)_2 = \frac{\alpha t}{L_2^2} = \frac{(8.4\times10^{-5})(60)}{(0.015)^2} = 22.4$$

$$\frac{1}{Bi} = \frac{k}{hL_2} = \frac{200}{(250)(0.015)} = 53.3$$

$$\left(\frac{\theta_o}{\theta_i}\right)_2 = 0.65$$

$$\left(\frac{\theta_o}{\theta_i}\right)_2 = 0.65 \quad \text{for the second slab}$$

Hence for the product

$$\left(\frac{\theta_o}{\theta_i}\right) = \left(\frac{\theta_o}{\theta_i}\right)_1 \left(\frac{\theta_o}{\theta_i}\right)_2$$

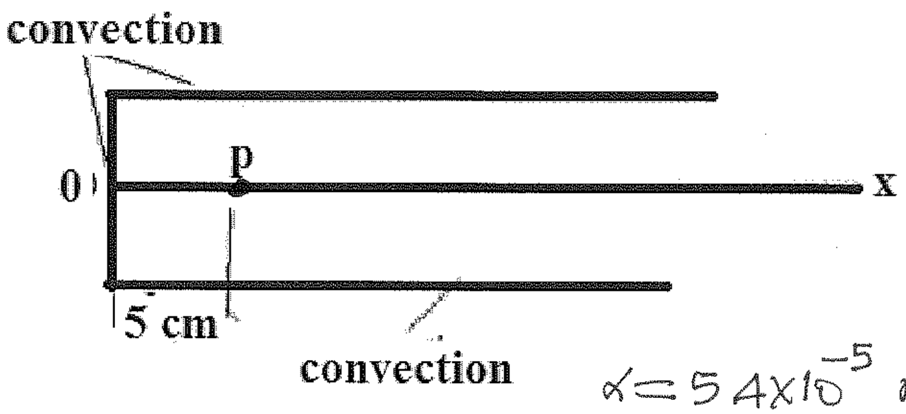
$$= (0.85)(0.65) = 0.55$$

$$0.55 = \frac{T_0 - T_\infty}{T_i - T_\infty} \Rightarrow T_0 = T_\infty + 0.55(T_i - T_\infty)$$

$$= 25 + 0.55(175 - 25)$$

$$= 107.5\,°C$$

8) A semi-infinite strip, $0<x<\infty$, $0<y<10$ cm, of fireclay brick [$k=1$ W/(m.°C), and $\alpha=$ ~~5.4×10^{-5}~~ 5.4×10^{-7} m²/s] is initially at a uniform temperature $T_i = 340$ °C. Suddenly all surfaces are subjected to convection, with a heat transfer coefficient $h=100$ W/(m².°C) into an ambient at $T_\infty = 40$ °C. Calculate the temperature T_o of a point P located along the midplane at a distance L= 5 cm from the surface as shown in figure, t= 2h after the start of the cooling.



$\alpha = 5.4\times10^{-5}$ m²/s

$k = 1$ W/m°C

Answer: 51.4 °C

$T_i = 340$°C $\quad T_\infty = 40$°C $\quad \bar{h} = 100$ W/m² °C

$t = 2$ hrs $\quad$ Point P: $\quad x = 5$ cm $\quad y = 5$cm

1) We have a slab of thickness $2L = 10$ cm subjected to convection

$L = 10/2 = 0.05$m

$$Fo = \frac{\alpha t}{L^2} = \frac{5.4\times10^{-7}\times7200}{(0.05)^2} = 1.56 \quad \left(\frac{\theta}{\theta_0}\right) = 0.095$$

$$\frac{1}{Bi} = \frac{k}{hL} = \frac{1}{(100)(0.05)} = 0.2$$

2) we have a semiinfinite body subjected to convection

$$\xi = \frac{x}{2\sqrt{\alpha t}} = \frac{0.05}{2\sqrt{(5.4\times10^{-7})(7200)}} = 0.4$$

h

$$\frac{h\sqrt{\alpha t}}{K} = \frac{100\sqrt{(5.4\times10^{-7})(7200)}}{1} = 6.2$$

$$\therefore\ 1-\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_2 = 0.6 \Rightarrow \left(\frac{T-T_\infty}{T_i-T_\infty}\right)_2 = 0.4$$

or

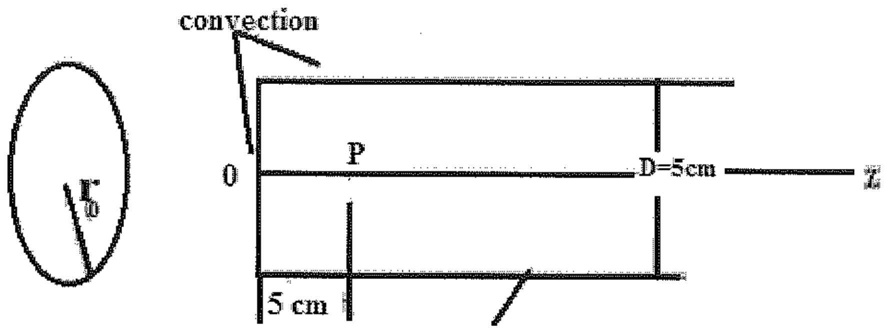
or $\left(\frac{\theta}{\theta_i}\right)_2 = 0.4$

Product solution

$$\left(\frac{\theta}{\theta_i}\right) = \left(\frac{\theta}{\theta_i}\right)_2\left(\frac{\theta}{\theta_i}\right)_1 = (0.4)(0.095) = 0.038$$

$$T = T_\infty + 0.038(T_i - T_\infty)$$
$$= 40 + 0.038(340-40) = 51.4\,°C$$

9) A semi-infinite, cylindrical iron bar [$\alpha = 1.6\times10^{-5}\ m^2/s$ and $k=60$ W/(m.°C)] of diameter D=5 cm, confined to the region $0 \le x \le \infty$, is initially at a uniform temperature $T_i=330$ °C. Suddenly the surfaces are subjected to convection with a heat transfer coefficient h= 200 W/(m².°C) into an ambient at $T_\infty = 30$ °C. Determine the temperature T_o of a point P located along the axis L= 3 cm from the flat surface t= 2 min after the start of the cooling. See figure given below.



$\alpha = 1.6\times10^{-5}\ m^2/s$

$k = 60\ W/m°C$ $\quad D = 5\ cm$ $\quad T_i = 330°C$ $\quad h = 200\ W/m^2°C$

$t = 2\ min = 60s \times 2 = 120s$

1) infinite cylinder

$r_o = 2.5\ cm = 0.025\ m$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{(1.6\times10^{-5})(120)}{(0.025)^2} = 3.1$$

$$\frac{1}{Bi} = \frac{k}{h r_o} = \frac{60}{(200)(0.025)} = 12$$

$$\left(\frac{\theta}{\theta_o}\right)_1 = 0.62$$

2) semi-infinite body

$$\xi = \frac{x}{2\sqrt{\alpha t}} = \frac{0.03}{2\sqrt{(1.6\times10^{-5})(120)}} = 0.34$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{200\sqrt{(1.6\times10^{-5})(120)}}{60} = 0.15$$

$$1 - \frac{T - T_\infty}{T_i - T_\infty} = 0.08$$

$$\left(\frac{\theta}{\theta_i}\right)_2 = 1 - 0.08 = 0.92$$

$$\left(\frac{\theta}{\theta_i}\right) = \left(\frac{\theta}{\theta_i}\right)_1 \left(\frac{\theta}{\theta_i}\right)_2 = 0.62)(0.92) = 0.57$$

$(T - T_\infty)/(T_i - T_\infty) = 0.57 \rightarrow T = 201°C$