CANKAYA UNIVERSITY
FACULTY OF ENGINEERING
MECHANICAL ENGINEERING DEPARTMENT
ME 313 HEAT TRANSFER

CHAPTER-5

EXAMPLES

1) A steel ball (c = 0.46 kJ/kg.°C, k = 35 W/m. °C) 5.0 cm in diameter and initially at a uniform temperature of 450°C is suddenly placed in a controlled environment in which the temperature is maintained at 100°C. The convection heat transfer coefficient is 10 W/m².°C. Calculate the time required for the ball to attain a temperature of 150°C.

$c = 0.46\ kJ/kg\,K$ $\quad T_i = 450°C$

$k = 35\ W/m\,°C$ $\quad T_\infty = 100°C$

$D = 5\,cm$ $\quad \bar{h} = 10\ W/m^2\,°C$

$$L_c = \frac{\forall}{A_s} = \frac{\frac{4}{3}\pi r_o^3}{4\pi r_o^2} = \frac{r_o}{3} = (2.5)/100 = 0.025\,m$$

$$Bi = \frac{\bar{h}L_c}{k} = \frac{(10)(0.025)}{35} = 0.00714 < 0.1$$

Use Lumped Model

$$\frac{\bar{h}A}{\rho c \forall} = 3.34 \times 10^{-4}\ 1/s$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{\bar{h}A}{\rho c \forall}\right)t\right]$$

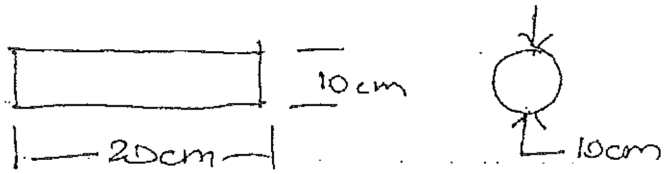
$t = 5826\ s$

Example-2

A copper cylinder 10 cm diameter, 20 cm long is removed from liquid nitrogen bath at $-196\,°C$ and exposed to air at $25°C$ with convection coefficient of $20\ W/m^2K$. Find the time required by cylinder to attain the temperature of $-110°C$. Take thermophysical properties as

$c = 380\ J/kgK$; $\rho = 8800\ kg/m^3$; $k = 360\ W/mK$

solution



$$L_c = s = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o^2 + 2\pi r_o L} = 0.02\ m$$

$$Bi = \frac{hs}{k} = \frac{(20)(0.02)}{360} = 1.11 \times 10^{-3} < 0.1$$

Lumped capacity model can be used

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{ht}{\rho c s}\right]$$

$$\frac{-110 - 25}{-196 - 25} = \exp\left[\frac{-20t}{8800 \times 0.02 \times 380}\right]$$

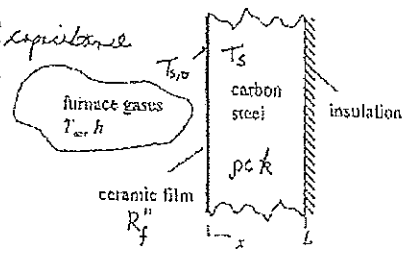
$t = 1648$ sec

Example - 3

A plane wall of a furnace is fabricated from plain carbon steel (k = 60 W/m·K, ρ = 7850 kg/m^3, c = 430 J/kg·K) and is of thickness L = 10 mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film. The thermal resistance of the coating per unit surface area is 0.01 m^2·K/W. The opposite surface is well insulated from the surroundings. At the furnace start-up the wall is at an initial uniform temperature of 300 K. The combustion gases enter the furnace at 1300K providing a convection coefficient of 25 W/m^2·K at the ceramic film. Assume the film has negligible thermal capacitance. How long will it take for the inner surface of the steel to achieve a temperature of 1200K? What is the temperature of the exposed surface of the ceramic film at this time?

assume: constant properties
negligible film thermal capacitance
neglect radiation

$L_c = L$



$$T_\infty \;\underset{1/h}{\text{—}\!\!\wedge\!\!\wedge\!\!\text{—}}\; T_{s,o} \;\underset{R_f''}{\text{—}\!\!\wedge\!\!\wedge\!\!\text{—}}\; T_s$$

(a) find t when $T_s = 1200$ K :

- use overall heat transfer coefficient to account for the thermal resistance of the ceramic film:

$$U(\text{per area}) = \frac{1}{\sum R_t} = \frac{1}{\frac{1}{h} + R_f''} = \frac{1}{\frac{1}{25} + 0.01} = 20 \frac{W}{m^2 \cdot K}$$

- modify the Biot number to account for the overall coefficient:

$$Bi = \frac{U \delta}{k} = \frac{(20)(0.01)}{60} = 0.0033 < 0.1$$

- use lumped capacitance method:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{U A_s}{\rho c \forall}\right) t\right] = \exp\left[-\left(\frac{U}{\rho c \delta}\right) t\right]$$

where $\delta = \frac{\forall}{A_s}$

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$T_i = 300\ K$

$T_\infty = 1300\ K$

$$\frac{(1200-1300)\,K}{(300-1300)\,K} = \exp\left[-\frac{20\,\frac{W}{m^2 K}}{(7850\,\frac{kg}{m^3})(430\,\frac{J}{kg\cdot K})(0.01\,m)}\,t\right]$$

$$0.1 = \exp\left(-0.0005925\,s^{-1}\cdot t\right)$$

$$\therefore\ t = 3886\,s = 1.08\ hr$$

(b) $T_{s,o}$: use thermal network

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R_f''$$

$$(25)(1300 - T_{s,o}) = (T_{s,o} - 1200)/(0.01)$$

$$\therefore\ T_{s,o} = 1220\ K$$

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4) A 50 mm thick iron plate is initially at 225°C. Its both surfaces are suddenly exposed to an environment at 25°C with convection coefficient of 500 W/m².K.
a) Calculate the center temperature, 2 minutes after the start of exposure.
b) Calculate the temperature at the depth of 10 mm from the surface, after 2 minutes of exposure.
c) Calculate the energy removed from the plate per square meter during this period. Take thermophysical properties of iron plate: k = 60 W/m.K, ρ = 7850 kg/m³, c = 460 J/kg, α = 1.6×10^{-5} m²/s.

$k = 60$ W/m K $\alpha = 1.6 \times 10^{-5}$ m²/s
$\rho = 7850$ kg/m3 $c = 460$ J/kg K

Solution

Given : A hot thick iron plate exposed to convection ambient on both surfaces

$2L = 50$ mm or $L = 25$ mm $= 0.025$ m, $k = 60$ W/m.K,
$T_i = 225°C$, $T_\infty = 25°C$, $t = 2$ min $= 120$ s,
$\rho = 7850$ kg/m³, $C = 460$ J/kg.K, $h = 500$ W/m².K
$\alpha = 1.6 \times 10^{-5}$ m²/s, Depth = 10 mm from the surface.

To find :

1. The centreline temperature of the plate, after 2 minute of exposure.
2. The temperature at the depth of 10 mm from the surface, after 2 minute.
3. Heat transferred during 2 minute.

Assumptions :

1. The heat transfer area of 1 m².
2. Constant properties.

Analysis : 1. Consider the plate of thickness 2L, hence considering L as characteristic length

The Biot No. $\quad Bi = \frac{hL}{k} = \frac{500 \times 0.025}{60} = 0.21$

The Biot no. is greater than 0.1, hence the lumped heat system analysis cannot be used. Using the Heisler charts :

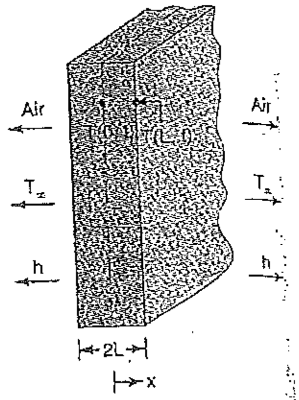


Fig. 6.31. Schematic of thick iron plate

Here $L_c = L$

$$\frac{1}{\text{Bi}} = \frac{1}{0.21} = 4.8$$

Fourier No. $\quad \text{Fo} = \frac{\alpha t}{L^2} = \frac{(1.6 \times 10^{-5}\ \text{m}^2/\text{s}) \times (120\ \text{s})}{(0.025\ \text{m})^2} = 3.07$

From Heisler chart for centreline temperature, for 1/Bi = 4.8 and Fo = 3.07

$$\frac{\theta_c}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.58$$

or $\quad T_0 = 0.58 \times (225 - 25) + 25 = 141°\text{C}.$ **Ans.**

2. Temperature at the depth of 10 mm from the surface,

$$x = L - \text{depth} = 25\ \text{mm} - 10\ \text{mm} = 15\ \text{mm}$$

Hence $\quad \frac{x}{L} = \frac{15}{25} = 0.6$

From chart for position temperature, for 1/Bi = 4.8 and $x/L = 0.6$

Temperature ratio at the location,

$$\frac{\theta}{\theta_c} = \frac{T - T_\infty}{T_0 - T_\infty} = 0.95$$

or $\quad T = 25 + 0.95 \times (141 - 25) = 135.2°\text{C}.$ **Ans.**

3. Heat loss from the plate during 2 minute exposure ;

$$\text{Bi} = 0.21$$

$$\text{Bi}^2\ \text{Fo} = (0.21)^2 \times 3.07 = 0.135$$

From the Gröber chart for heat transfer ratio for plane wall

$$Q/Q_i = 0.45$$

where $\quad Q_i = \rho V C(T_i - T_\infty) = \rho(A\ 2L)\ C(T_i - T_\infty)$

$= (7850\ \text{kg/m}^3) \times (1\ \text{m}^2 \times 0.05\ \text{m}) \times (460\ \text{J/kg}) \times (225 - 25)(\text{K})$

$= 35.33 \times 10^6\ \text{J/m}^2 = 35.33 \times 10^3\ \text{kJ/m}^2$

The heat transferred during 2 minute,

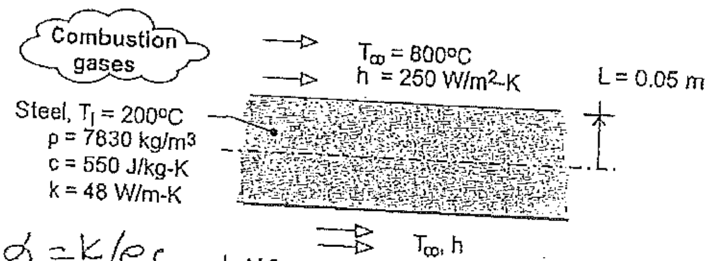
$$Q = 0.45 \times 35.33 \times 10^3\ \text{kJ/m}^2 = 15.9 \times 10^3\ \text{kJ/m}^2.$$ **Ans.**

5) Annealing is a process by which steel is reheated and then cooled to make is less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830\ kg/m^3$, $c = 550\ J/kg.K$, $k = 48\ W/m.K$) which is initially at a uniform temperature of T_i = 200°C and is to be heated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace, where products of combustion at T_∞ = 800°C maintain a convection coefficient of h = 250 W/m².K on both surfaces of the plate. How long should the plate be left in the furnace?

KNOWN: Thickness, properties and initial temperature of steel slab. Convection conditions.

FIND: Heating time required to achieve a minimum temperature of 550°C in the slab.

SCHEMATIC:



$\alpha = k/\rho c = 1.115 \times 10^{-5}\ m^2/s$

ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible radiation effects, (3) Constant properties.

ANALYSIS: With a Biot number of $hL/k = (250\ W/m^2 \cdot K \times 0.05m)/48\ W/m \cdot K = 0.260$, a lumped capacitance analysis should not be performed. At any time during heating, the lowest temperature in the slab is at the midplane, [illegible]

$L_c = L$ $\quad Bi = 0.26 > 0.1$

$$\frac{\theta_o}{\theta_i} = \theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{550 - 800}{200 - 800} = 0.417$$

$$\frac{1}{Bi} = 3.84$$

$F_o = ?$

$\therefore \quad F_o = 3.84$

$$F_o = \frac{\alpha t}{L^2}$$

$$t = \frac{L^2}{\alpha} F_o = \frac{(0.05)^2 (3.84)}{1.115 \times 10^{-5}} = 860\ sec$$

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Method 2:

$$\theta^* = C_1 \exp(-\xi_1^2 Fo) \cos(\xi_1 x^*)$$

At $x=0$ $x^*=0$ plate center

$$\theta_o^* = c_1 \exp(-\xi_1^2 Fo)$$

$$-Fo = \frac{1}{\xi_1^2} \ln(\theta_o^*/C_1)$$

now for $Bi=0.26$ $\xi_1 = 0.4801$

$C_1 = 1.0382$

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{550-800}{200-800} = 0.417$$

$$Fo = \frac{1}{(0.4801)^2} \ln\left(\frac{0.417}{1.0382}\right) \approx 3.95$$

$$t = \frac{L^2}{\alpha} Fo = \frac{(0.05)^2 (3.95)}{(1.115 \times 10^{-5})} \cong 885 \text{ sec}$$

6) A plate of stainless steel (18% Cr, 8% Ni) (k=16.3 W/m.°C, α=0.44x10^{-5} m²/s) has a thickness of 3.0 cm and is initially uniform in temperature at 500°C. The plate is suddenly exposed to a convection environment on both sides at 40°C with h = 150 W/m².°C. Calculate the times for the center and face temperatures to reach 120°C.

$L_c = L \qquad Bi = 0.1381 > 0.1$

$L = 0.015\text{ m} \qquad T_i = 500°\text{C} \qquad T_\infty = 40°\text{C} \qquad h = 150\ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$

$k = 16.3\ \frac{\text{W}}{\text{m}\cdot°\text{C}} \qquad \alpha = 0.44\times10^{-5}\ \text{m}^2/\text{s} \qquad \frac{k}{hL} = \frac{6.3}{(150)(0.015)} = 7.24$

Part a) time for center temperature

$$\left.\begin{array}{l}\frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{120-40}{500-40} = 0.174 \\ \frac{1}{Bi} = \frac{k}{hL} = 7.24\end{array}\right] \quad Fo = \frac{\alpha t}{L^2} = 13.9 \text{ Fig}$$

$$t = \frac{L^2}{\alpha} Fo = \frac{(0.015)^2 (13.9)}{0.44\times10^{-5}} \approx 711 \text{ sec}$$

b) Surface temperature $T = 120°c$

Fig

$$\left.\begin{array}{l}\frac{x}{L} = 1 \\ \frac{1}{Bi} = \frac{k}{hL} = 7.24\end{array}\right] \quad \frac{\theta}{\theta_o} = 0.93$$

Now we know that

$$\left(\frac{\theta}{\theta_i}\right) = \left(\frac{\theta_o}{\theta_i}\right)\left(\frac{\theta}{\theta_o}\right)$$

Since $T = 120°C$ ← Surface temperature

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \frac{120-40}{500-40} = 0.174$$

$\therefore \quad 0.174 = \left(\frac{\theta_o}{\theta_i}\right)(0.93)$

$\Rightarrow \quad \frac{\theta_o}{\theta_i} = 0.174/0.93 = 0.187$

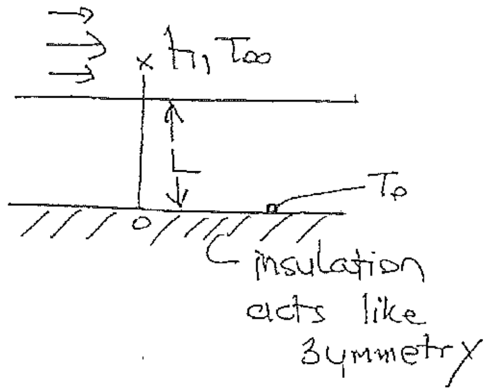
$\therefore$ From Fig

$\left.\begin{array}{l} \frac{\theta_o}{\theta_i} = 0.187 \\ \frac{1}{Bi} = 7.24 \end{array}\right] F_o = \frac{\alpha t}{L^2} = 13$

$t = 665\ s$

7) A slab of copper (k=370 W/m.°C, α=11.23x10^{-5} m^2/s) having a thickness of 3.0 cm is initially at 300°C. It is suddenly exposed to a convection environment on the top surface at 80°C while the bottom surface is insulated. In 6 min the surface temperature drops to 140°C. Calculate the value of convection heat transfer coefficient

$L_c = L$



$\alpha = 11.23 \times 10^{-5}$

$k = 370\ W/m^2\,°C$

$L = 3\ cm$

$T_i = 300°C$

$T_\infty = 80°C$

$t = 6\ min \rightarrow T = 140°C$

↳ surface temperature

From Fig.

$$\left.\begin{array}{l} F_o = \tau = \dfrac{\alpha t}{L^2} = 44.92 \\[2ex] \dfrac{1}{Bi} = \dfrac{k}{hL} = \dfrac{370}{0.03h} \end{array}\right] \quad \frac{\theta_o}{\theta_i} = ?$$

T_o and h unknown

from

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hL} \\[2ex] \dfrac{x}{L} = 1 \end{array}\right] \Rightarrow \frac{\theta}{\theta_o}$$

We will follow iterative solution

see next page

we will assume $\dfrac{1}{Bi} = \dfrac{k}{hL}$ (actually we are assuming h) from fig

$$\left.\begin{array}{l}\frac{1}{Bi} = \left(\frac{k}{hL}\right) = 100 \\ \tau = \frac{\alpha t}{L^2} = 44.92\end{array}\right] \Rightarrow \frac{\theta_o}{\theta_i} = 0.65$$

from

$$\left.\begin{array}{l}\frac{1}{Bi} = 100 \\ \frac{X}{L} = 1\end{array}\right] \quad \frac{\theta}{\theta_o} = 1$$

$$\frac{\theta}{\theta_i} = \left(\frac{\theta_o}{\theta_i}\right)\left(\frac{\theta}{\theta_o}\right)$$

$$\frac{140-80}{300-80} \neq (0.65)(1)$$

so assumed h is not correct

b) Assume $\frac{k}{hL} = \frac{1}{Bi} = 40$

$$\left.\begin{array}{l}\frac{1}{Bi} = 40 \\ \tau = 44.92\end{array}\right] \quad \frac{\theta_o}{\theta_i} = 0.34$$

$$\left.\begin{array}{l}\frac{1}{Bi} = 40 \\ \frac{X}{L} = 1\end{array}\right] \quad \frac{\theta}{\theta_o} = 0.98$$

$$\frac{\theta}{\theta_i} = \left(\frac{\theta_o}{\theta_i}\right)\left(\frac{\theta}{\theta_o}\right)$$

$$0.25 = (0.34)(0.98) \approx 0.33$$

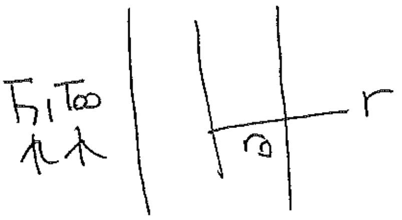
Pretty close, not bad

TAKE $\frac{k}{hL} \cong 40$ $\quad h = \frac{370}{40(0.03)} \approx 310 \ W/m^2\,°C$

8) An infinitely cylinder (k=17 W/m.°C, ρ=8000 kg/m³, c=0.42 kJ/kg. °C) 5.0 cm in a diameter and initially at 550 °C is suddenly exposed to a convection environment at 50°C and h = 340 W/m².°C. Calculate the center and surface tempeature of the cylinder and the heat lost per unit length 2 min after the cylinder is exposed to the environment.

$L_c = r_0/2 \qquad Bi = \frac{\bar{h} r_0}{2k} = \frac{(340)(2.5/100)}{2 \times 17} = 0.25 > 0.1$

use charts



a) $F_0 = \frac{\alpha t}{r_0^2} \cong 1$

$Bi = \frac{\bar{h} r_0}{k} = 0.5$

$\left.\right\} \frac{\theta_0}{\theta_i} = \frac{T_0 - T_\infty}{T_i - T_\infty} \cong 0.47$

$T_0 = 50 + (0.47)(550 - 50) = 285\,°C$

b) use position correction chart

$\frac{r}{r_0} = 1$

$\frac{1}{Bi} = \frac{k}{\bar{h} r_0} = 2$

$\left.\right\} \frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} \cong 0.37$

$\theta = 0.37\,\theta_0$

$T = T_0 + 0.37(T_0 - T_\infty)$

$= 285 + 0.37(285 - 50) \cong 372\,°C$

9) A solid iron rod ($\alpha = 2\times10^{-5}\ m^2/s$, $k = 60\ W/m^{\circ}.C$) of diameter D = 6 cm, initially at temperature T_i = 800°C, is suddenly dropped into oil bath at T_∞ = 50°C. The heat transfer coefficient between the fluid and solid surface is h = 400 W/m².°C.

a) Determine centerline temperature after 10 minutes.

b) How long will it take the centerline temperature to reach 100°C?

$$L_c = r_o/2 = \frac{3}{2} = 1.5\ cm \qquad L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o L} = r_o/2$$

$$Bi = \frac{hL_c}{K} = 0.1 \leftarrow \text{use charts}$$

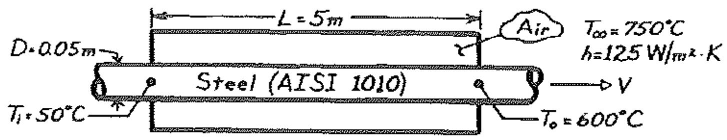
$$\left.\begin{aligned} F_o &= \frac{\alpha t}{r_o^2} = 13.3 \\ \frac{1}{Bi} &= \frac{K}{hr_o} = 5 \end{aligned}\right\} \frac{T_o - T_\infty}{T_i - T_\infty} = 0.006$$

$$\therefore T_o = 50 + 0.006(800-50) = 54.5\ ^{\circ}C$$

b) $$\left.\begin{aligned} \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{100-50}{800-50} = 0.067 \\ \frac{1}{Bi} &= 5 \end{aligned}\right\} F_o = 7.7$$

$$F_o = \frac{\alpha t}{r_o^2} \Rightarrow t = \frac{F_o r_o^2}{\alpha} = 347\ s.$$

10) Cylindrical steel rods (AISI 1010), 50 mm in diameter, are heat treated by drawing them through an oven 5 m long in which air is maintained at 750°C. The rods enter at 50°C and achieve a centerline temperature of 600°C before leaving. For a convection coefficient of 125 W/m².K, estimate the speed at which the rods must be drawn through the oven.



AISI 1010 steel (at $\bar{T} = 600K$) $k = 48.8 \frac{W}{mK}$

$\rho = 7832$ kg/m³, $c_p = 434$ J/kg K

$\alpha = 4.44 \times 10^{-5}$ m²/s

$L_c = \frac{r_o}{2}$ $\quad Bi = \frac{(\bar{h})(r_o/2)}{K} = 0.032 < 0.1$

Use Lumped method.

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\quad\right)t\right]$$

$$\frac{\bar{h}A}{\rho c \forall} = \frac{(\bar{h})(2\pi r_o L)}{\rho c (\pi r_o^2) L} = \frac{2\bar{h}}{\rho c r_o} = \frac{(2)(125)}{(7832)(434)(\frac{25}{1000})}$$

$$= 0.002941$$

$$\frac{600-750}{50-750} = \ln(-0.002941\, t) \Rightarrow t \cong 524\,s$$

$$V = \frac{L}{t} = \frac{5m}{524s} = 0.00954 \text{ m/s}$$

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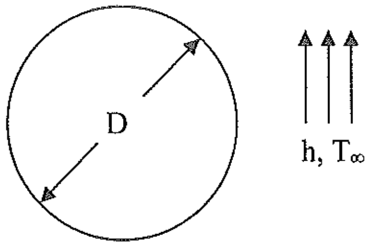
11) Oranges are usually refrigerated as a preservative measure. However, some people prefer to eat oranges that are a little cooler than room temperature but not as cold as the refrigerator makes them. Determine the time it takes for an orange removed from a refrigerator to reach 20°C.



Use the following conditions:

Refrigerated temperature = 4°C
Ambient room temperature = 23°C
Surface conductance = *6* W/(m2.K)
Thermal conductivity of an orange = 0.431 W/(m.K)*
Density of orange = 998 kg/m3
Specific heat of orange = 2 kJ/(kg.K)
Orange diameter = 105 mm

$$Bi_c = \frac{\bar{h} L_c}{K} = \frac{\bar{h} \forall}{KA} = \frac{\bar{h}\, \pi D^3/6}{K \pi D^2} = \frac{\bar{h} D}{6K}$$

$$= \frac{6(0.105)}{6(0.431)} = 0.2437 > 0.1 \quad \text{Use charts}$$

$$Bi = \frac{\bar{h} r_0}{K} = 0.731 \rightarrow 1/Bi = 1.36$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.158$$

$$F_0 = \frac{\alpha t}{r_0^2} = 1.05$$

$$\alpha = \frac{k}{\rho c} = 2.159 \times 10^{-7} \ m^2/s$$

$$\therefore t = 1.34 \times 10^4 s \approx 3.7 \text{ hours}$$

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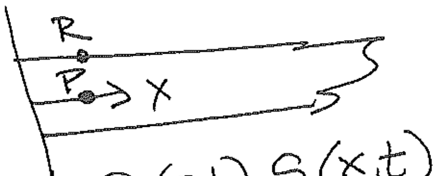
Heat transfer:

$$Bi^2 Fo = (0.371)^2 (1.05) = 5.61 \times 10^{-1}$$
$$Bi = \frac{h r_o}{k} = 0.731$$

$$\frac{Q}{Q_o} = 0.7$$

$$Q = 0.7 (\rho c V)(T_i - T_\infty)$$
$$= (0.7)(998)(2000)\left(\frac{\pi}{6}\right)(0.105)^3 (4-23)$$
$$= -1.609 \times 10^{4}\, J$$ ← heat is transferred to orange

12) A semi-infinite aluminum cylinder (k=215 W/m.°C, $\alpha=8.4\times10^{-5}$ m²/s) 5 cm in diameter is initially at uniform temperature of 200°C. It is suddenly subjected to a convection boundary condition at 70°C with h = 525 W/m².°C. Calculate the temperatures at the axis and surface of the cylinder 10 cm from the end 1 min after exposure to the environment.



$C(r,t)\ S(x,t)$

a) Semi infinite plate

$$\frac{h\sqrt{\alpha t}}{k} = \frac{525\sqrt{8.4\times10^{-5}\times60}}{215} = 0.173$$

$$\frac{x}{2\sqrt{\alpha t}} = \frac{(10/100)}{2\sqrt{(8.4\times10^{-5})(60)}} = 0.7042$$

$$\therefore \quad \frac{T-T_i}{T_\infty - T_i} = 1 - \frac{T-T_\infty}{T_i - T_\infty} \cong 0.036$$

$$\therefore \quad \left(\frac{\theta}{\theta_i}\right)_{\text{semi infinite solid}} = 1 - 0.036 = 0.964$$

for infinite cylinder:

$r_o = 2.5\,cm$

$$\left.\begin{aligned} \frac{k}{h r_o} &= 16.38 \\ \frac{\alpha t}{r_o^2} &= 8.064 \end{aligned}\right\} \quad \frac{\theta_o}{\theta_i} = 0.38 \leftarrow \text{center temperature ratio of cylinder}$$

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Surface temperature ratio

$$\left.\begin{array}{l}\frac{1}{Bi} = 16.38 \\ \frac{r}{r_0} = 1\end{array}\right\} \quad \frac{\theta}{\theta_0} = 0.97$$

for cylinder center : $C = \left(\frac{\theta}{\theta_i}\right)_{cyl} = 0.38$

for " " surface $C = (\theta/\theta_i)_{cyl} = (0.38)(0.97)$
$= 0.3686$

At point P;

$$\left(\frac{\theta}{\theta_i}\right)_{\substack{semi\\ infinite\\ cylinder}} = CS = (0.38)(0.964) = 0.366$$

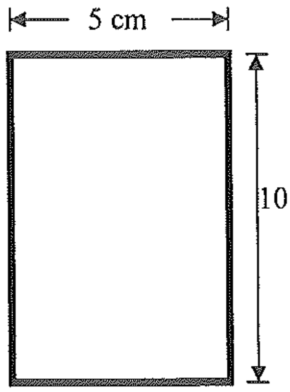
$\therefore$ At $r = 0$ $\quad T = 70 + 0.366(200-70)$
$= 117.6°c$

at point R:

$$\left(\frac{\theta}{\theta_i}\right)_{\substack{semi\\ infinite\\ cylinder}} = CS = (0.3686)(0.964) = 0.356$$

$$T = 70 + 0.356(200-70)$$
$$= 116.3°c$$

13) A short aluminum cylinder (k=215 W/m.°C, α=8.4x10^{-5} m^2/s) 5.0 cm in diameter and 10 cm long is initially at uniform temperature of 200°C. It is suddenly subjected to a convection environment at 70°C, and h = 525 W/m^2.°C. Calculate the temperature at a radial position of 1.25 cm and a distance of 0.625 cm from one end of the cylinder 1 min after exposure to the environment.

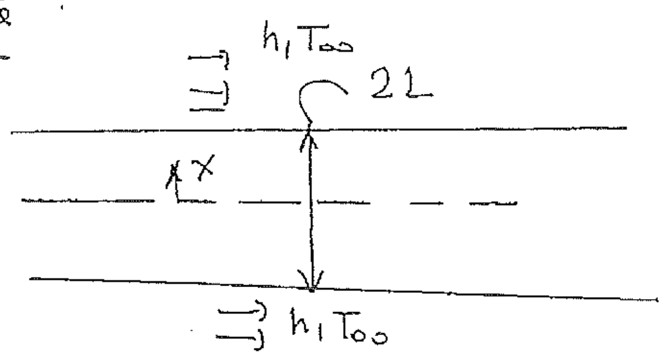


$\alpha = 8.4 \times 10^{-5}\ m^2/s$

$k = 215\ W/m°C$

infinite cylinder   infinite plate = short cylinder $\Rightarrow$ $P(x,t)\ C(r,t)$

<u>infinite plate</u> ·



$L = 5$ cm

$X = 5 - 0.625 = \quad 4.375$ cm

$\frac{X}{L} = 4.375/5 = 0.875$

$\left.\begin{array}{l} \frac{k}{hL} = 8.19 \\ \frac{\alpha t}{L^2} = 2.016 \end{array}\right\} \quad \frac{\theta_o}{\theta_i} = 0.75$

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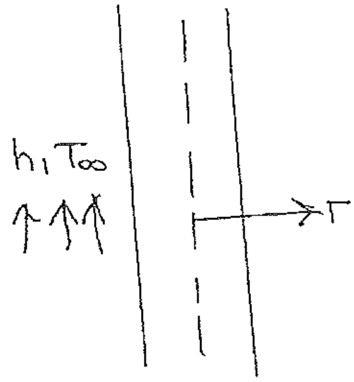
$$\left.\begin{aligned}\frac{1}{Bi} &= \frac{k}{hL} = 8.19\\ \frac{x}{L} &= 0.875\end{aligned}\right\}\quad \frac{\theta}{\theta_0} = 0.95$$

$$\therefore\quad P = \left(\frac{\theta}{\theta_i}\right)_{plate} = 0.75)(0.95) = 0.7125$$

infinite cylinder



$$r_0 = 2.5\ \text{cm}$$

$$\frac{r}{r_0} = \frac{1.25}{2.5} = 0.5$$

$$\left.\begin{aligned}\frac{k}{hr_0} &= 16.38\\ \frac{\alpha t}{r_0^2} &= 8.064\end{aligned}\right]\quad \frac{\theta_0}{\theta_i} = 0.38$$

$$\left.\begin{aligned}\frac{1}{Bi} &= \frac{k}{hr_0} = 16.38\\ \frac{r}{r_0} &= 0.5\end{aligned}\right\}\quad \frac{\theta}{\theta_0} = 0.98$$

$$C = \left(\frac{\theta}{\theta_i}\right)_{cyl} = (0.38)(0.98) = 0.3724$$

Combining solutions for plate and cylinder

$$\left(\frac{\theta}{\theta_i}\right)_{\substack{short\\cylinder}} = (0.7125)(0.3724) = 0.265$$

$$T = T_\infty + 0.265\,(T_i - T_\infty)$$

$$= 70 + 0.265\,(200 - 70) = 104.5\,^\circ C$$

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Calculate the heat loss from short cylinder

plate:

$$\left.\begin{aligned}\frac{hL}{k} &= 0.122\\ \frac{h^2\alpha t}{k^2} &= 0.03\end{aligned}\right\} \quad \frac{Q}{Q_0} = 0.22$$

cylinder: $r_0 = 2.5\,cm = 0.025\,m$

$$\left.\begin{aligned}\frac{hr_0}{k} &= 0.061\\ \frac{h^2\alpha t}{k^2} &= 0.03\end{aligned}\right\} \quad \frac{Q}{Q_0} = 0.55$$

$$\left(\frac{Q}{Q_0}\right)_{total} = 0.22 + 0.55\,(1-0.22) = 0.649$$

i.e

$$\left(\frac{Q}{Q_0}\right)_{tot} = \left(\frac{Q}{Q_0}\right)_1 + \left(\frac{Q}{Q_0}\right)_2 \left[1 - \left(Q/Q_0\right)\right]$$

$$Q_0 = \rho c V \theta_i = (2707)(0.896)\pi(0.025)^2(0.1)(200-7$$
$$= 61.9 \text{ kJ}$$

$$Q = (61.9)(0.649) = 40.2 \text{ kJ}$$