Cankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 313 Heat Transfer

Chapter 9
Example Solutions

1. A large vertical plate 4.0 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer if the plate is 10 m wide.

We first determine the film temperature as:

$$T_f = \frac{60+10}{2} = 35°C = 308\ K$$

The properties of interest are thus:

$$\beta = \frac{1}{308} = 3.25\times10^{-3}\ K^{-1} \qquad k = 0.02685\ W/mK$$

$$\nu = 16.5\times10^{-6}\ m^2/s \qquad Pr = 0.7$$

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}Pr = \frac{(9.8)(3.25\times10^{-3})(60-10)(4)^3}{(16.5\times10^{-6})^2}(0.7)$$

$$\Rightarrow Ra_L = 2.62\times10^{11}$$

We then obtain $\overline{Nu}_L$:

$$\overline{Nu}_L = \left[0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2$$

$$\Rightarrow \overline{Nu}_L = \left[0.825 + \frac{(0.387)(2.62\times10^{11})^{1/6}}{\left[1+(0.492/0.7)^{9/16}\right]^{8/27}}\right] \Rightarrow \overline{Nu}_L = 716$$

The heat transfer coefficient is then:

$$\bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{(716)(0.02685)}{(4.0)} \Rightarrow \bar{h} = 4.80 \ W/m^2 °C$$

The heat transfer is: $q = \bar{h} A (T_s - T_\infty) = (4.80)(4)(10)(60-10) = 9606 \ W$

2. A horizontal pipe 1 ft (0.3048 m) in diameter is maintained at temperature of 250°C in a room where the ambient air is at 15°C. Calculate the free convection heat loss per meter of length.

We first determine the Rayleigh number and then select the approciate constants from Table 9-1. The properties of air are evaluated at the film temperature

$$T_f = \frac{T_s + T_\infty}{2} = \frac{250+15}{2} = 132.5\,°C = 405.5\,K$$

$k = 0.03406\ W/m°C$ $\quad$ $\beta = \frac{1}{T_f} = \frac{1}{405.5} = 2.47\times10^{-3}\ K^{-1}$

$\nu = 26.54\times10^{-6}\ m^2/s$ $\quad$ $Pr = 0.687$

$$Ra_D = Gr_D\,Pr = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}Pr = \frac{(9.8)(2.47\times10^{-3})(250-15)(0.3048)^3(0.687)}{(26.54\times10^{-6})^2}$$

$\Rightarrow Ra_D = 1.571\times10^8$

$\overline{Nu_D}$ is defined as: $\overline{Nu_D} = C\,Ra_D^{\,n}$

From Table 9.1, $C = 0.125$, $n = 0.333$ so that:

$$\overline{Nu_D} = (0.125)(1.571\times10^8)^{0.333} = 67.03$$

$$\overline{Nu_D} = \frac{\bar{h}D}{k} \Rightarrow \bar{h} = \frac{k\overline{Nu_D}}{D} = \frac{(0.03406)(67.03)}{(0.3048)} = 7.49\ W/m^2°C$$

The heat transfer per unit length is then calculated from:

$$\frac{q}{L} = \bar{h}\pi D(T_s - T_\infty) = (7.49)\,\pi\,(0.3048)(250-15) = 1685.4\ W/m = 1.69\ kW/m$$

3. A fine wire having a diameter of 0.02 mm is maintained at a constant temperature of 54 °C by an electric current. The wire is exposed to air at 1 atm and 0 °C. Calculate the electric power necessary to maintain the wire temperature if the length is 50 cm.

The film temperature is $T_f = (54+0)/2 = 27\,°C = 300\,K$,

So the properties are:

$\beta = 1/300 = 0.00333\ K^{-1}$ $\qquad \nu = 15.69 \times 10^{-6}\ m^2/s$

$k = 0.02624\ W/m°C$ $\qquad Pr = 0.708$

The Rayleigh number is calculated as:

$$Ra_D = Gr_D Pr = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} Pr$$

$$\Rightarrow Ra_D = \frac{(9.8)(0.00333)(54-0)(0.02\times10^{-3})^3}{(15.69\times10^{-6})^2}(0.708) = 4.05\times10^{-5}$$

$\overline{Nu}_D$ is defined as:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C Ra_D^n$$

From Table 9.1 we find $C = 0.675$ and $n = 0.058$ so that:

$$\overline{Nu}_D = (0.675)(4.05\times10^{-5})^{0.058} \Rightarrow \overline{Nu}_D = 0.375$$

$$\Rightarrow \bar{h} = \overline{Nu}_D\left(\frac{k}{D}\right) = \frac{(0.375)(0.02624)}{(0.02\times10^{-3})} \Rightarrow \bar{h} = 492.6\ W/m^2°C$$

The heat transfer or power required is then:

$$q = \bar{h}A(T_s - T_\infty) = (492.6)\pi(0.02\times10^{-3})(0.5)(54-0)$$

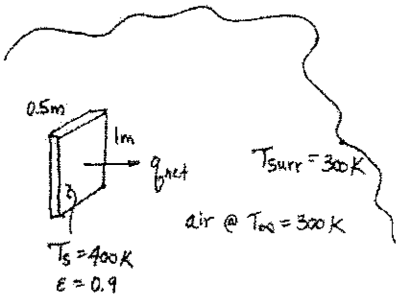
$$\Rightarrow q = 0.836\ W$$

Example-4

A flat, isothermal heating panel, 0.5 m wide by 1 m high, is mounted to one wall of a large room. The surface of the panel has an emissivity of 0.90 and is maintained at 400 K. If the walls and air in the room are at 300 K, what is the net rate at which heat is transferred from the panel to the room?

assume:

1-D transfer from panel (isothermal)
neglect conduction through panel
constant properties of air @ $T_f = 350\,K$
large enclosure (radiation)
panel is a gray surface ($\alpha = \varepsilon$)
steady state



analysis:

surface energy balance,

$$\cancel{\dot{E}_{st}} = \dot{E}_{in} + \cancel{\dot{E}_g} - \dot{E}_{out}$$

$$0 = q - (q_{conv} + q_{rad,net})$$

$$\therefore \quad q = hA_s(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{surr}^4)A_s$$

to find h: determine flow conditions

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr$$

(A.4 @ $T_f = 350K$: $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.03\ \frac{W}{m \cdot K}$, $Pr = 0.7$)

assume air is an ideal gas: $\beta = \frac{1}{T}$

$$Ra_L = \frac{(9.81\ m/s^2)\left(\frac{1}{350\ K}\right)(400-300)\ K\ (1m)^3}{(20.92 \times 10^{-6})^2}(0.7) = 4.483 \times 10^9$$

$Ra_L > Ra_c \quad \therefore$ turbulent flow

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{(0.387)(4.483\times10^9)^{1/6}}{\left[1+(0.492/0.7)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{(0.387)(4.483\times10^9)^{1/6}}{\left[1+(0.492/0.7)^{9/16}\right]^{8/27}}\right\}^2 = 195.6$$

$$\bar{h} = (195.6)(0.03\ \mathrm{W/m\cdot k})/(1\mathrm{m}) = 5.87\ \mathrm{W/m^2\cdot K}$$

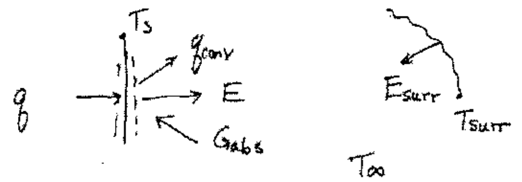
$$q = \Big[\underbrace{(5.87\,\tfrac{\mathrm{W}}{\mathrm{m^2\cdot K}})(400-300)\mathrm{K}}_{q''_{conv} = 586.7\ \mathrm{W/m^2}} + \underbrace{(0.9)(5.67\times10^{-8}\,\tfrac{\mathrm{W}}{\mathrm{m^2 K^4}})(400^4-300^4)\mathrm{K}}_{q''_{rad} = 893.0\ \mathrm{W/m^2}}\Big]\underbrace{(1\mathrm{m})(0.5\mathrm{m})}_{A_s}$$

$\therefore \quad q = 740\ \mathrm{W}$

Comments:

Often radiation is comparable to free convection, if not more significant.

aside:



$E = \varepsilon\sigma T_s^4$

$G_{abs} = \alpha G = \alpha\sigma T_{surr}^4$

if surface is "gray": $\alpha = \varepsilon$

$$q = q_{conv} + E - G_{abs} = q_{conv} + \underbrace{\varepsilon\sigma T_s^4 - \varepsilon\sigma T_{surr}^4}_{q_{rad,net}}$$