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Lecture 11

Numerical Integration

Integration

The general form of a definite integral (also called an antiderivative) is:

$$I(f) = \int_a^b f(x)dx$$

where $f(x)$, called the integrand, is a function of the independent variable x, and a and b are the limits of the integration. The value of the integral $I(f)$ is a number when a and b are numbers. Graphically, as shown in Fig. 9-4, the value of the integral corresponds to the shaded area under the curve of $f(x)$ between a and b.

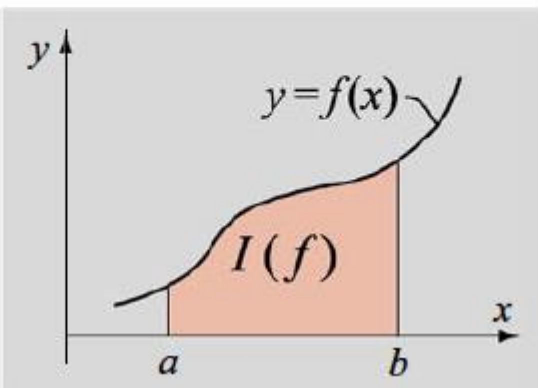


Figure Definite integral of $f(x)$ between a and b.

Trapezoidal Method

$$I(f) \approx \frac{h}{2}[f(a) + f(b)] + h\sum_{i=2}^{N} f(x_i)$$

Simpson's (1/3) method

$$I(f) \approx \frac{h}{3}\left[f(a) + 4\sum_{i=2,4,6}^{N} f(x_i) + 2\sum_{j=3,5,7}^{N-1} f(x_j) + f(b)\right]$$

where $h = (b-a)/N$.

- The subintervals must be ***equally spaced***.
- The ***number of subintervals*** within $[a, b]$ ***must be an <u>even</u> number***.

Simpson's (1/3) method

$$I(f) \approx \frac{3h}{8}\left[f(a) + 3\sum_{i=2,5,8}^{N-1}[f(x_i) + f(x_{i+1})] + 2\sum_{j=4,7,10}^{N-2} f(x_j) + f(b)\right]$$

Equation is ***Simpson's 3/8 method*** for numerical integration. Simpson's 3/8 method can be used if the following two conditions are met:

- The subintervals are ***equally spaced.***
- The ***number of subintervals*** within $[a, b]$ ***must be divisible by 3***.

Example

Calculate the integral of function

$$f(x) = x^3 sin(x)$$

between zero and one .Use N points

```
>> N=500;
x=linspace(0,1,N);
h=x(2)-x(1);
f=(x.^3).*sin(x);
 I=(h/2)*(f(1)+f(N))+h*(sum(f))
I =
   0.1788
```

USE OF MATLAB BUILT-IN FUNCTIONS FOR INTEGRATION

The quad *command*

The form of the quad command is:

```
I = quad(function,a,b)
```

The value of the integral.

The function to be integrated.

The integration limits.

- The function can be entered as a string expression, or as a function handle.
- The function $f(x)$ must be written for an argument x that is a vector (use element-by-element operations), such that it calculates the value of the function for each element of x.

- The user has to make sure that the function does not have a vertical asymptote (singularity) between a and b.
- `quad` calculates the integral with an absolute error that is smaller than 1.0×10^{-6}. This number can be changed by adding an optional `tol` argument to the command:

```
q = quad(function,a,b,tol)
```

`tol` is a number that defines the maximum error. With larger `tol` the integral is calculated less accurately but more quickly.

The `quadl` command:

The form of the `quadl` (the last letter is a lower case L) command is exactly the same as the `quad` command:

```
I = quadl(function,a,b)
```

The value of the integral.

The function to be integrated.

The integration limits.

All the comments listed above for the `quad` command are valid for the `quadl` command. The difference between the two commands is in the numerical method used for calculating the integration. The `quadl` uses the adaptive Lobatto method.

The `trapz` *command*

The built-in function `trapz` can be used for integrating a function that is given as discrete data points. It uses the trapezoidal method of numerical integration. The form of the command is:

```
q = trapz(x,y)
```

where `x` and `y` are vectors with the x and y coordinates of the points, respectively. The two vectors must be of the same length.

The `dblquad` *command*

The built-in function `dblquad` can be used to evaluate a double integral. The format of the command is:

```
I = dblquad(function, xmin, xmax, ymin, ymax)
```

The value of the integral.

The function to be integrated.

The integration limits.

- The function can be entered as a string, or as a function handle.
- The function $f(x, y)$ must be written for an argument x that is a vector (use element-by-element operations) and for an argument y that is a scalar.

- The limits of integration are constants.

Example

Use numerical integration to calculate the following integral:

$$\int_0^8 (xe^{-x^{0.8}} + 0.2)dx$$

```
>> f=@(x) x.*exp(-x.^(0.8))+0.2
f =
   @(x)x.*exp(-x.^(0.8))+0.2

>> I=quad(f,0,8)
I =
   3.1604
```

b) trapz

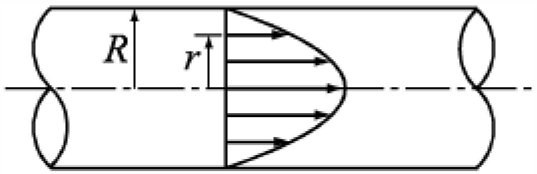
```
>> x=[0:0.1:8];
>>  f=@ (x) x.*exp(-x.^(0.8))+0.2;
>> F=f(x);
>> I=trapz(x,F)
I =
    3.1596
```

Example

. The flow rate Q (volume of fluid per second) in a round pipe can be calculated by:

$$Q = \int_0^r 2\pi v r dr$$

For turbulent flow the velocity profile can be estimated by: $v = v_{max}\left(1 - \frac{r}{R}\right)^{1/n}$. Determine Q for $R = 0.25$ in., $n = 7$, $v_{max} = 80$ in./s.



Method: quad

```
>> vmax=80; R=0.25; n=7;
>> F=@ (r) 2*pi*vmax*(1-r/R).^(1/n).*r;
>> Q=quad(F,0,R)
Q =
  12.8282
```

b) trapz

```
>>  vmax=80; R=0.25; n=7;
F=@ (r) 2*pi*vmax*(1-r/R).^(1/n).*r;
r=0:0.01:0.25;
f=F(r);
trapz(r,f)
ans =
   12.5192
```