1) Steady-state temperature distribution in a one-dimensional wall of thermal conductivity 50 W/m.K and thickness 50 mm is observed to be

\[ T(x) = a + bx^2 \]

where \( a = 200 \, ^\circ C, b = -200 \, ^\circ C / m^2 \) and \( x \) is in meters and \( T \) is in \( ^\circ C \).

(a) What is the heat generation rate \( \dot{q} \) in the wall?
(b) Determine heat fluxes at the two wall faces.
Solution

**KNOWN:** Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

**FIND:** (a) The heat generation rate, \( \dot{q} \), in the wall, (b) Heat fluxes at the wall faces and relation to \( \dot{q} \).

**SCHEMATIC:**

![Schematic Diagram]

**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

**ANALYSIS:** (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.21 re-written as

\[
\dot{q} = -k \frac{d}{dx} \left[ \frac{dT}{dx} \right]
\]

Substituting the prescribed temperature distribution,

\[
\dot{q} = -k \frac{d}{dx} \left[ \frac{d}{dx} \left( a + bx^2 \right) \right] = -k \frac{d}{dx} \left[ 2bx \right] = -2bk
\]

\[
\dot{q} = 2\left(2000^\circ\text{C/m}^2\right) \times 50 \text{ W/m} \cdot \text{K} = 2.0 \times 10^5 \text{ W/m}^3.
\]

(b) The heat fluxes at the wall faces can be evaluated from Fourier’s law,

\[
\dot{q}_x(x) = -k \frac{d}{dx} T(x).
\]

Using the temperature distribution \( T(x) \) to evaluate the gradient, find

\[
\dot{q}_x(x) = -k \frac{d}{dx} \left[ a + bx^2 \right] = -2k bx.
\]

The fluxes at \( x = 0 \) and \( x = L \) are then

\[
\dot{q}_x(0) = 0
\]

\[
\dot{q}_x(L) = -2kbL = -2 \times 50 \text{ W/m} \cdot \text{K} \left( 2000^\circ\text{C/m}^2 \right) \times 0.050 \text{m}
\]

\[
\dot{q}_x(L) = 10,000 \text{ W/m}^2.
\]

**COMMENTS:** From an overall energy balance on the wall, it follows that, for a unit area,

\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = 0 \quad \dot{q}_x(0) - \dot{q}_x(L) + \dot{q}L = 0
\]

\[
\dot{q} = \frac{\dot{q}_x(L) - \dot{q}_x(0)}{L} = \frac{10,000 \text{ W/m}^2 - 0}{0.050 \text{m}} = 2.0 \times 10^5 \text{ W/m}^3.
\]
2) A house has a composite wall of wood, fiberglass insulation, and plaster board, as indicated in the sketch. On a cold winter day, the convection heat transfer coefficients are $h_o = 60 \text{ W/m}^2\text{K}$ and $h_i = 30 \text{ W/m}^2\text{K}$. The total wall surface area is 350 m$^2$.

Determine the total heat flux through the wall.

Solution

**PROPERTIES:** Table A-3, \( \bar{T} = (T_i + T_o) / 2 = (20 -15)^\circ \text{C}/2 = 2.5^\circ \text{C} = 300 \text{K} \): Fiberglass blanket, 28 kg/m$^3$, $k_b = 0.038 \text{ W/mK}$; Plywood siding, $k_s = 0.12 \text{ W/mK}$; Plasterboard, $k_p = 0.17 \text{ W/mK}$.

**ANALYSIS:** (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

\[
R_{\text{tot}} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}.
\]

(b) The total heat loss through the house wall is

\[
q = \Delta T / R_{\text{tot}} = (T_i - T_o) / R_{\text{tot}}.
\]

Substituting numerical values, find

\[
R_{\text{tot}} = \frac{1}{30 \text{ W/m}^2\text{K} \times 350 \text{ m}^2} + \frac{0.17 \text{ W/mK} \times 350 \text{ m}^2}{0.02 \text{ m}} + \frac{0.038 \text{ W/mK} \times 350 \text{ m}^2}{1} + \frac{0.12 \text{ W/mK} \times 350 \text{ m}^2}{0.17 \text{ W/mK} \times 350 \text{ m}^2} + \frac{60 \text{ W/m}^2\text{K} \times 350 \text{ m}^2}{60 \text{ W/m}^2\text{K} \times 350 \text{ m}^2}
\]

\[
R_{\text{tot}} = [9.52 + 16.8 + 75.2 + 47.6 + 4.76] \times 10^{-5} \text{ C/W} = 831 \times 10^{-5} \text{ C/W}
\]

The heat loss is then,

\[
q = [20 - (-15)]^\circ \text{C}/831 \times 10^{-5} \text{ C/W} = 4.21 \text{ kW}.
\]
3) A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 3 m below the surface. The pipe has an outer diameter of 1 m and is insulated with a layer of cellular glass 200 mm thick. What is the heat loss per unit length of pipe when heated oil at 110° C flows through the pipe and the surface of the earth is at a temperature of 10° C?

Solution

**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction through insulation, two-dimensional through soil, (3) Constant properties, (4) Negligible oil convection and pipe wall conduction resistances.

**PROPERTIES:** *Table A-3*, Soil (300K): $k = 0.52 \text{ W/m-K}$; *Table A-3*, Cellular glass (365K): $k = 0.069 \text{ W/m-K}$.

**ANALYSIS:** The heat rate can be expressed as

$$q = \frac{T_1 - T_2}{R_{\text{tot}}}$$

where the thermal resistance is $R_{\text{tot}} = R_{\text{ins}} + R_{\text{soil}}$. From Equation 3.33,

$$R_{\text{ins}} = \frac{\ln(D_2/D_1)}{2\pi L k_{\text{ins}}} = \frac{\ln(1.4/1)}{2\pi L \times 0.069} \text{ W/m-K} = \frac{0.776 \text{ m-K/W}}{L}.$$  

From Equation 4.21 and Table 4.1,

$$R_{\text{soil}} = \frac{1}{Sk_{\text{soil}}} = \frac{\cosh^{-1}(2z/D_2)}{2\pi L k_{\text{soil}}} = \frac{\cosh^{-1}(6/1.4)}{2\pi \times (0.52 \text{ W/m-K})L} = \frac{0.653 \text{ m-K/W}}{L}.$$  

Hence,

$$q = \frac{(110 - (-10))^\circ C}{\frac{1}{L}(0.776 + 0.653) \text{ m-K/W}} = 84 \text{ W/m} \times L$$

$$q' = \frac{q}{L} = 84 \text{ W/m}.$$
4) In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch.

To cure the bond at a temperature $T_0$, a radiant source is used to provide a heat flux $q''_0$ (W / m$^2$), all of which is absorbed at the bonded surface. The back of the substrate is maintained at $T_1$ while the free surface of the film is exposed to air at $T_\infty$ and a convection heat transfer coefficient $h$. Calculate the heat flux $q''_0$ (W / m$^2$) required to maintain the bonded surface at $T_0 = 60 \degree C$.

**Solution**

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux $q''_0$ is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.

(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q''_0 = q'_1 + q''_2$$

$$q''_0 = \frac{T_0 - T_\infty}{R_{cv}^* + R_1^*} + \frac{T_0 - T_1}{R_s^*}$$

where the thermal resistances are

$$R_{cv}^* = \frac{1}{h} = \frac{1}{50 \text{ W/m}^2 \cdot \text{K}} = 0.020 \text{ m}^2 \cdot \text{K/W}$$

$$R_1^* = \frac{L_f}{k_f} = \frac{0.00025 \text{ m}}{0.025 \text{ W/m} \cdot \text{K}} = 0.010 \text{ m}^2 \cdot \text{K/W}$$

$$R_s^* = \frac{L_s}{k_s} = \frac{0.001 \text{ m}}{0.05 \text{ W/m} \cdot \text{K}} = 0.020 \text{ m}^2 \cdot \text{K/W}$$

$$q''_0 = \frac{(60 - 20) \degree \text{C}}{[0.020 + 0.010] \text{m}^2 \cdot \text{K/W}} + \frac{(60 - 30) \degree \text{C}}{0.020 \text{m}^2 \cdot \text{K/W}} = \frac{(1333 + 1500) \text{W/m}^2}{2833 \text{W/m}^2} <$$